

32nd Nuclear Physics Workshop 2026, September 28–30, 2026

Systematic Study of Superheavy Nuclei within a Microscopic Collective Hamiltonian: Impact of Quantum Shape Fluctuations

Presenter : Rongyan Hu

Supervisor : Prof. Zhipan Li

Date : September 28–30, 2026

School of Physical Science and Technology, Southwest University

Outline

1. Introduction

2. Microscopic Collective Hamiltonian Model

3. Structure and Stability of Superheavy Nuclei

4. Summary and Outlook



Outline

1. Introduction

2. Microscopic Collective Hamiltonian Model

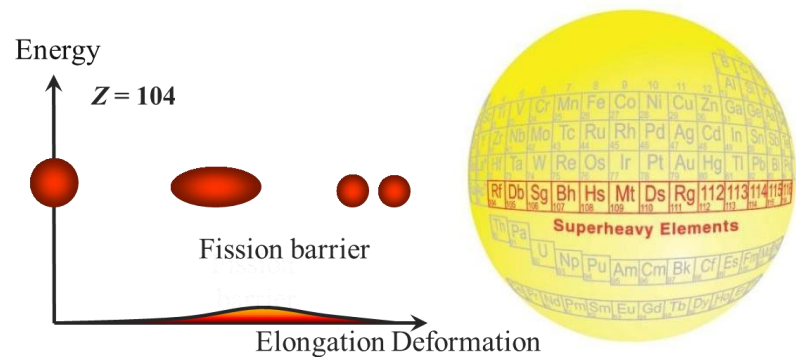
3. Structure and Stability of Superheavy Nuclei

4. Summary and Outlook



Experimental exploration of superheavy nuclei

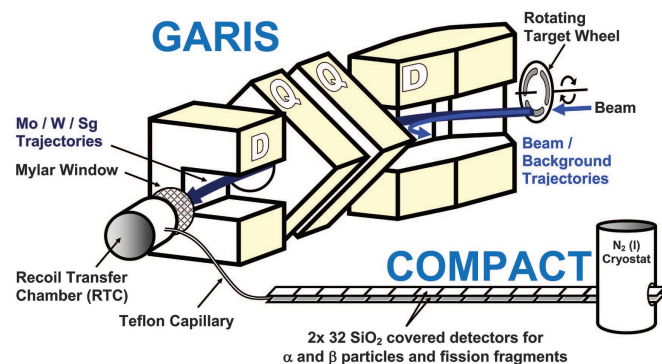
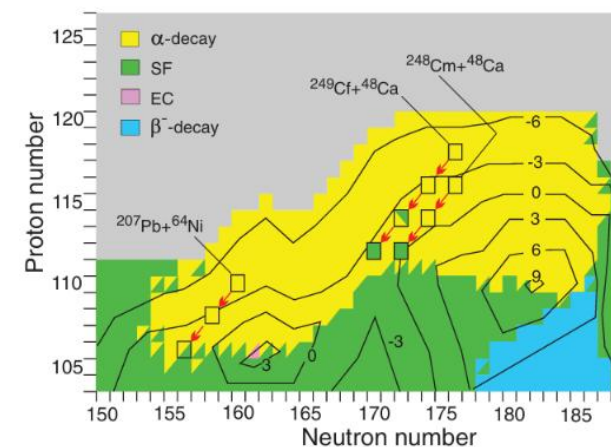
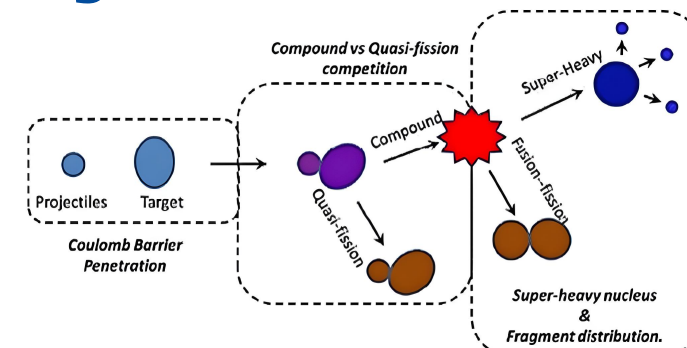
Zhou Shangui. *NPR*, 34(3), 318-331 (2017)



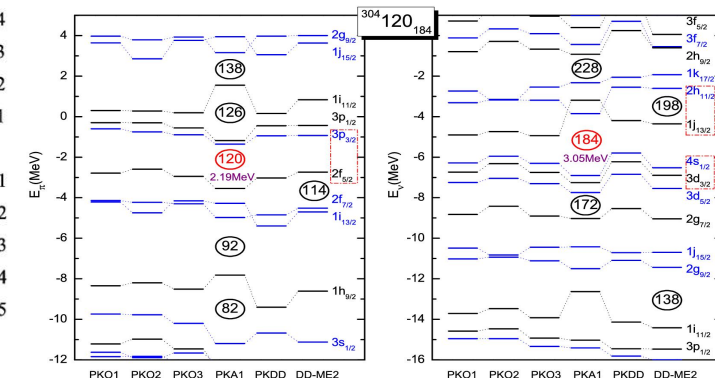
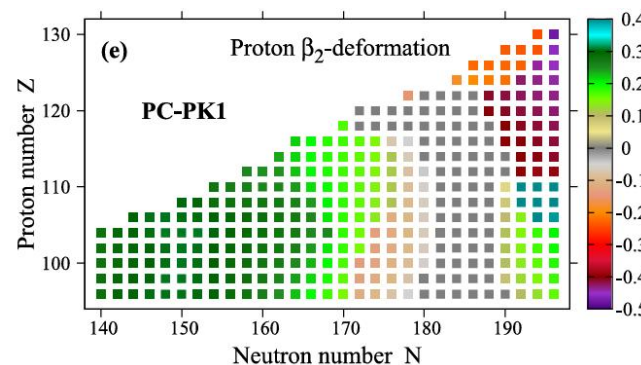
Chemical properties of superheavy elements

Synthesis of superheavy nuclei

Stability of superheavy nuclei: α -decay and spontaneous fission
Structure of superheavy nuclei: Magic numbers, deformation...



SCIENCE 345, 1491 (2014)



Experimental exploration of superheavy nuclei

□ Laboratory synthesis of superheavy nuclei via heavy-ion fusion-evaporation reactions ($Z = 118$, Og):

- HIRFL in Lanzhou, China
- Flerov Laboratory of Nuclear Reactions in Dubna, Russia
- GSI in Darmstadt, Germany
- Lawrence Berkeley National Laboratory, USA
- Lawrence Livermore National Laboratory, USA
- RIKEN in Wako, Japan
- GANIL in Caen, France

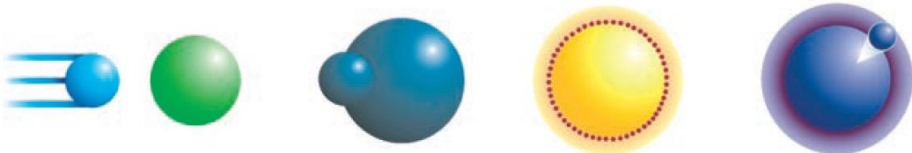
Hofmann_Münzenberg2000_RMP72-733

Morita...2004_JPSJ73-2593

Oganessian...2007_JPG34-R165

Oganessian...2010_PRL104-142502

Zhang...2012_CPL29-012502



Theoretical studies of superheavy nuclear structure

◆ Macroscopic-Microscopic Models

M. Brack et al., Rev. Mod. Phys. 44, 320 (1972).

P. Möller & J. R. Nix, J. Phys. G 20, 1681 (1994).

A. Sobiczewski & K. Pomorski, PPNP 58, 292 (2007).

◆ Density Functional Theory (DFT)

M. Bender et al., Rev. Mod. Phys. 75, 121 (2003).

J. Meng et al., PPNP 57, 470 (2006).

L. M. Robledo et al., J. Phys. G 46, 013001 (2019).

G. G. Adamian et al., Eur. Phys. J. A 57, 89 (2021).

N. Schunck & D. Regnier, PPNP 125, 103963 (2022).

◆ Superheavy nuclei: magic numbers, shell structure, deformations, and fission barriers

S. Ćwiok et al., Phys. Rev. Lett. 83, 1108 (1999).

S. Ćwiok et al., Nature 433, 705 (2005).

J. Erler et al., J. Phys. G 38, 033101 (2011).

S. A. Giuliani et al., Rev. Mod. Phys. 91, 011001 (2019).

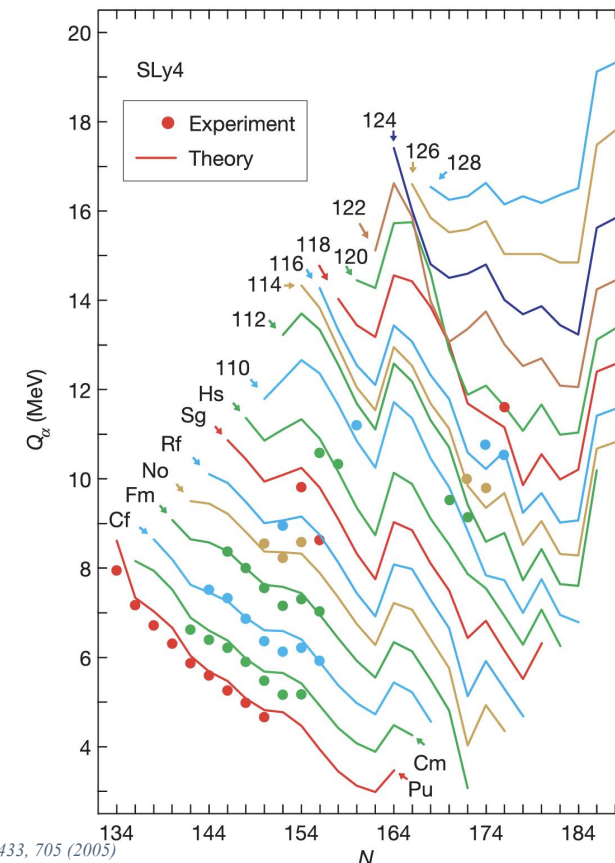
O. R. Smits et al., Nat. Rev. Phys. 6, 86 (2024).

R. Rodríguez-Guzmán et al., J. Phys. G 53, 075109 (2026).

Predictive power of nuclear DFT

➤ **Intrinsic ground state, deformation, PES, etc.**

➤ **A microscopic basis for collective dynamics**



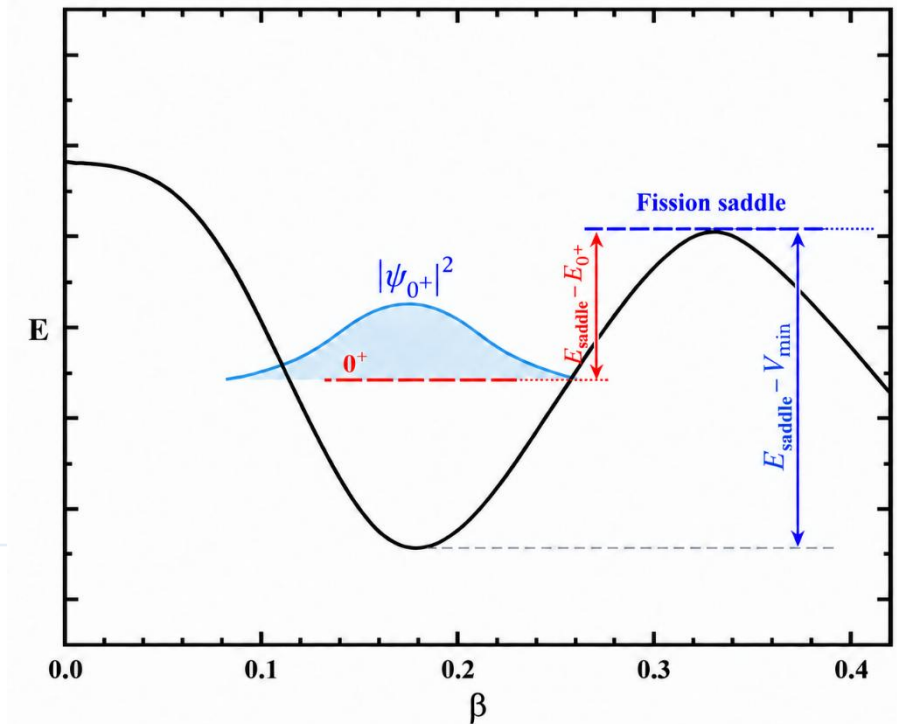
Ćwiok et al., Nature 433, 705 (2005)

Motivation

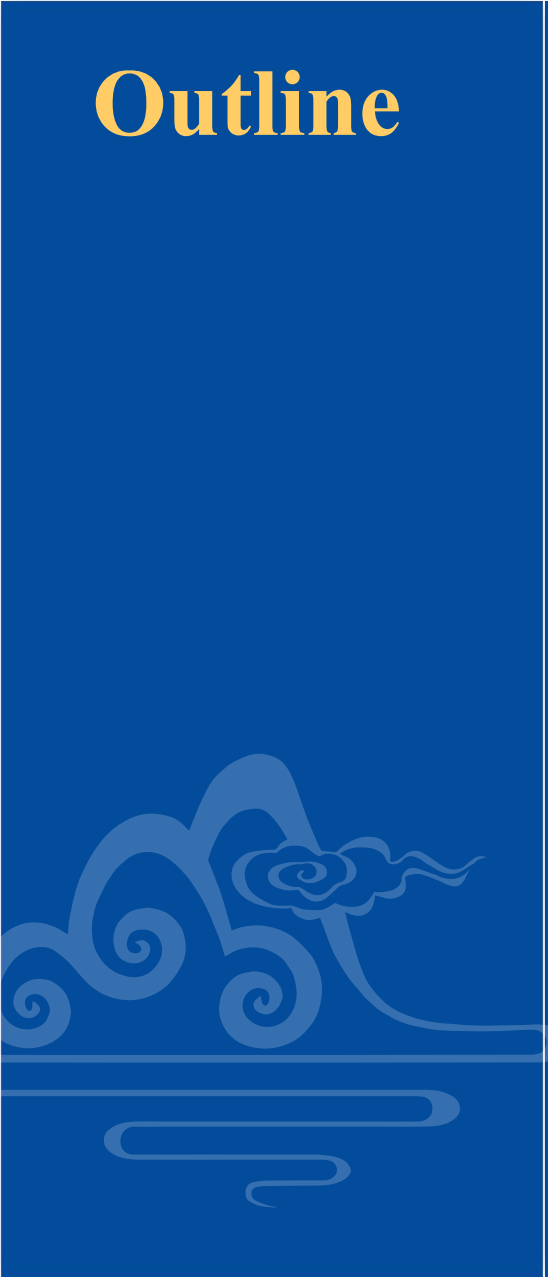
Our work: triaxial TRHB (PC-PK1) + microscopic 5DCH

- ◆ Rotational symmetry restoration
- ◆ Quantum shape fluctuations and mixing
- ◆ Triaxial collective dynamics
- ◆ Collective potential with zero-point-energy corrections

From potentials to collective states



Outline



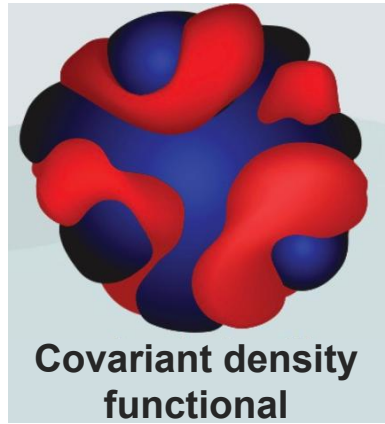
1. Introduction

2. Microscopic Collective Hamiltonian Model

3. Structure and Stability of Superheavy Nuclei

4. Summary and Outlook

Microscopic Collective Hamiltonian Model



Construct

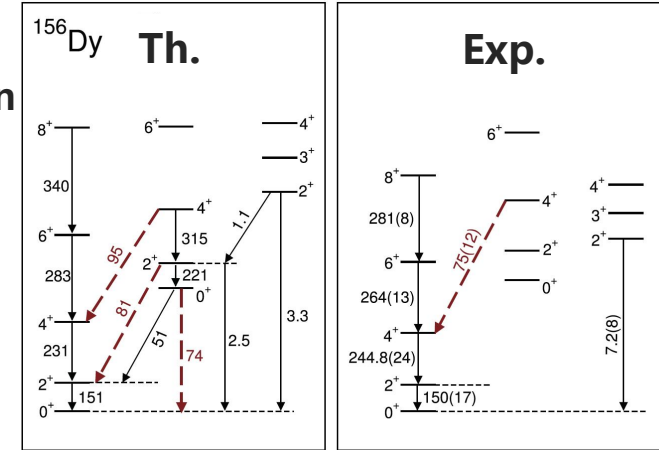


Microscopic
collective
parameters

**Microscopic collective
Hamiltonian**

$$\hat{H}_{\text{coll}} = \hat{T}_{\text{vib}} + \hat{T}_{\text{rot}} + V_{\text{coll}}(\beta_2, \beta_3, \gamma, \Omega \dots)$$

Solve the equation



$$\begin{aligned} E_{\text{CDF}} &= \int d^3(\mathbf{r}) \varepsilon_{\text{CDF}}(\mathbf{r}) \\ &= \sum_k \int d\mathbf{r} v_k^2 \bar{\psi}_k(\mathbf{r}) (-i\gamma \nabla + m) \psi_k(\mathbf{r}) \\ &+ \int d\mathbf{r} \left(\frac{\alpha_S}{2} \rho_S^2 + \frac{\beta_S}{3} \rho^3 + \frac{\gamma_S}{3} \rho_S^4 + \frac{\delta_S}{2} \rho_S \Delta \rho_S + \frac{\alpha_V}{2} j_\mu j^\mu + \frac{\gamma_V}{4} (j_\mu j^\mu)^2 \right. \\ &\left. + \frac{\delta_V}{2} j_\mu \Delta j^\mu + \frac{\alpha_{TV}}{2} \vec{j}_{TV}^\mu \cdot (\vec{j}_{TV})_\mu + \frac{\delta_{TV}}{2} \vec{j}_{TV}^\mu \cdot \Delta (\vec{j}_{TV})_\mu + \frac{e^2}{2} \rho_p A^0 \right) \end{aligned}$$

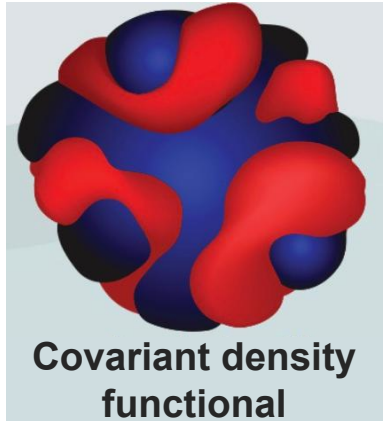
ZPLi, Niksic, Vretenar, Meng, Lalazissis, & Ring, PRC (2009)

ZPLi, Song, Yao, Vretenar & Meng, PLB (2013)

Tao, Zhao, ZPLi, Niksic & Vretenar, PRC (2017)

Xiang, ZPLi, Niksic, Vretenar & Long, PRC (2020)

Microscopic Collective Hamiltonian Model



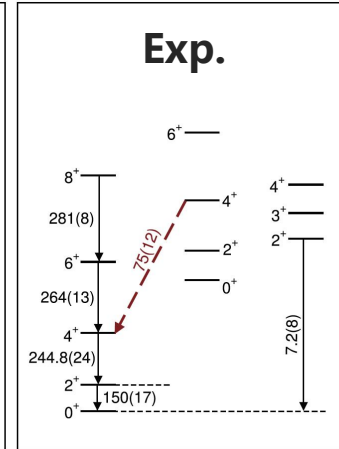
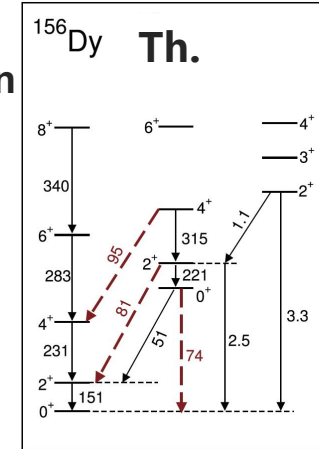
Construct
Microscopic collective parameters

Microscopic collective Hamiltonian

$$\hat{H}_{\text{coll}} = \hat{T}_{\text{vib}} + \hat{T}_{\text{rot}} + V_{\text{coll}}$$

$(\beta_2, \beta_3, \gamma, \Omega \dots)$

Solve the equation



$$\begin{aligned} \hat{H}_{\text{coll}} = & -\frac{\hbar^2}{2\sqrt{wr}} \left\{ \frac{1}{\beta^4} \left[\frac{\partial}{\partial \beta} \sqrt{\frac{r}{w}} \beta^4 B_{\gamma\gamma} \frac{\partial}{\partial \beta} - \frac{\partial}{\partial \beta} \sqrt{\frac{r}{w}} \beta^3 B_{\beta\gamma} \frac{\partial}{\partial \gamma} \right] \right. \\ & + \frac{1}{\beta \sin 3\gamma} \left[-\frac{\partial}{\partial \gamma} \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta\gamma} \frac{\partial}{\partial \beta} + \frac{1}{\beta} \frac{\partial}{\partial \gamma} \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta\beta} \frac{\partial}{\partial \gamma} \right] \Big\} \\ & + \frac{1}{2} \sum_{k=1}^3 \frac{\hat{J}_k^2}{\mathcal{I}_k} + V_{\text{coll}}(\beta, \gamma). \end{aligned}$$

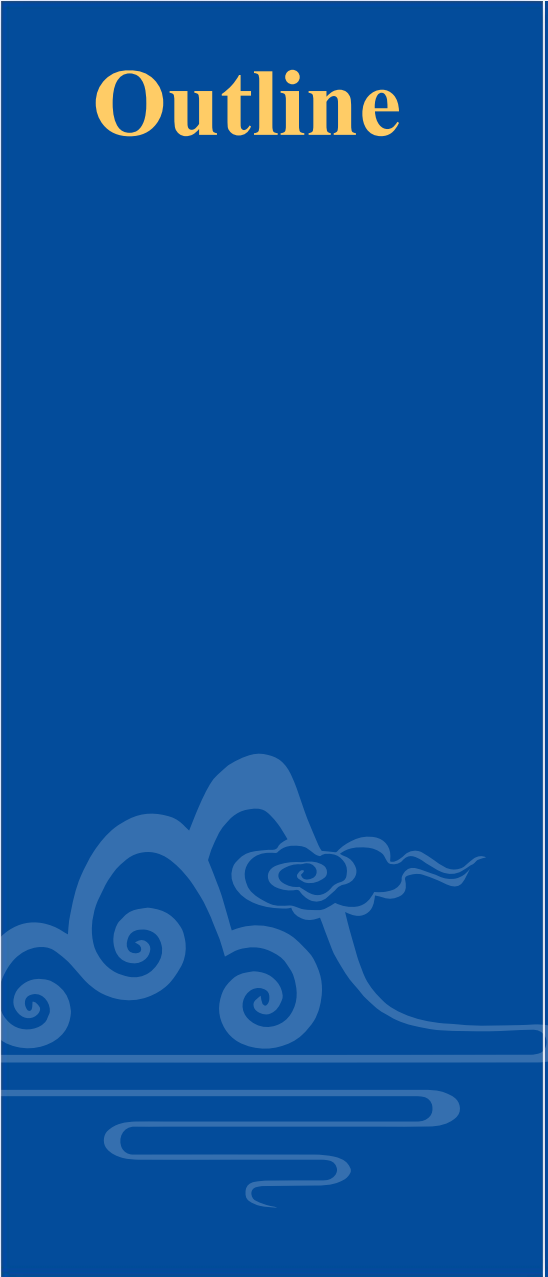
ZPLi, Niksic, Vretenar, Meng, Lalazissis, & Ring, PRC (2009)

ZPLi, Song, Yao, Vretenar & Meng. PLB (2013)

Tao, Zhao, ZPLi, Niksic & Vretenar, PRC (2017)

Xiang, ZPLi, Niksic, Vretenar & Long, PRC (2020)

Outline



1. Introduction

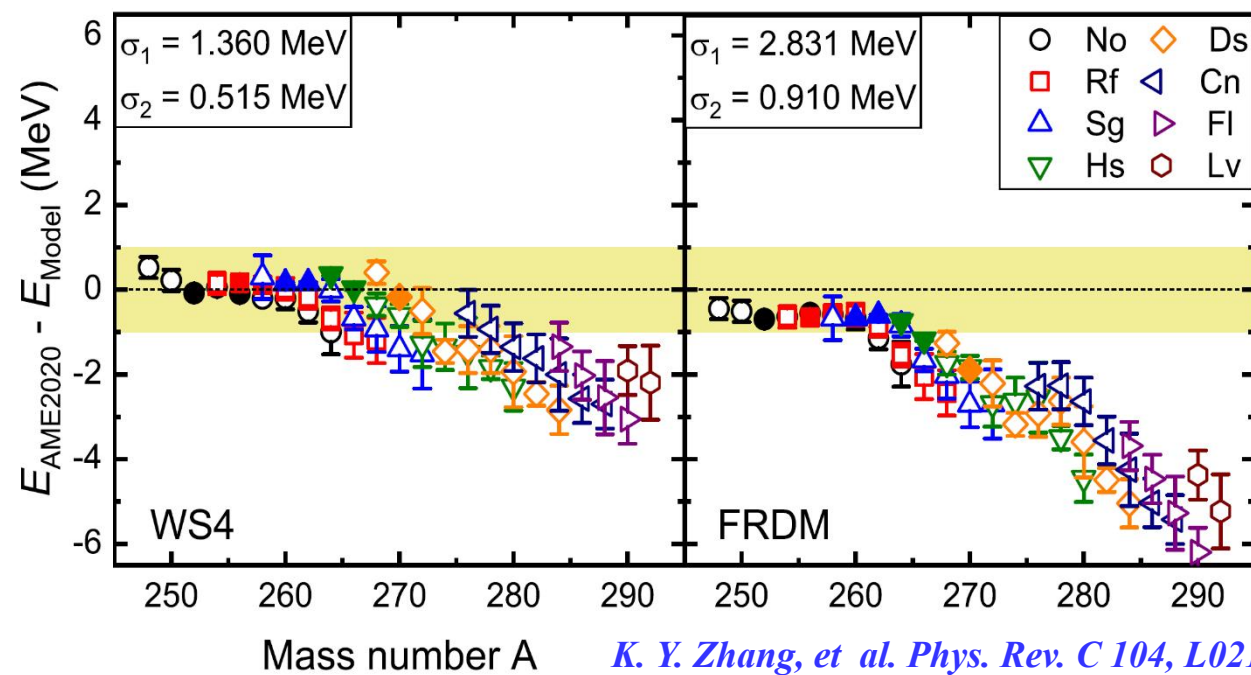
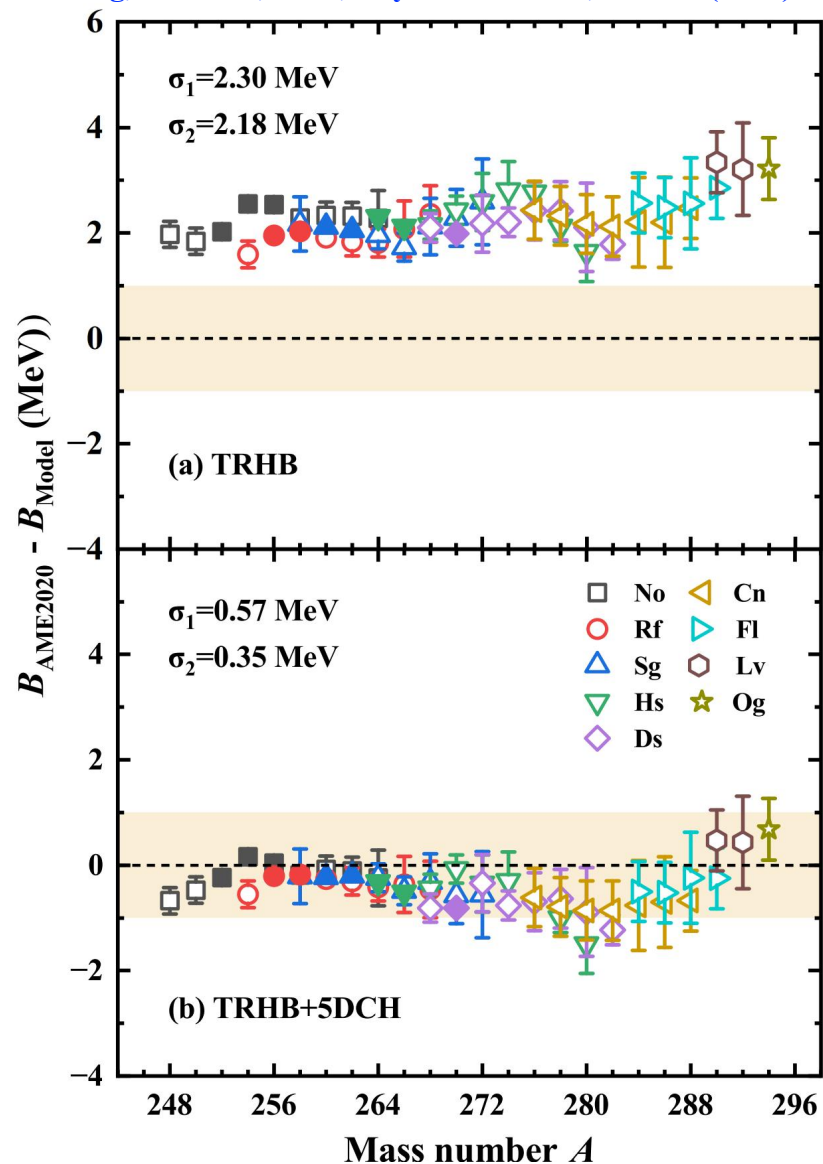
2. Microscopic Collective Hamiltonian Model

3. Structure and Stability of Superheavy Nuclei

4. Summary and Outlook

Validation I: masses of superheavy nuclei ($102 \leq Z \leq 118$)

X. Q. Yang, R. Y. Hu, et al., Phys. Lett. B 876, 140382 (2026).

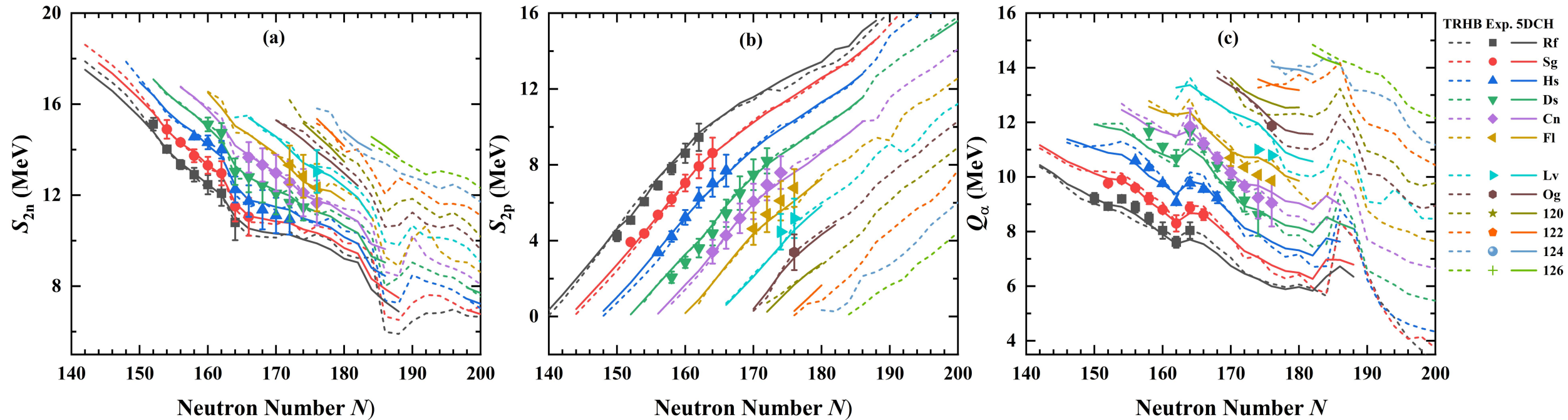


K. Y. Zhang, et al. Phys. Rev. C 104, L021301 (2021).

- Dynamical correlation energies are **included self-consistently**;
- The collective Hamiltonian model based on the **PC-PK1** functional provides an excellent description;
- **TRHB + 5DCH** yields more reliable **isospin-dependent behavior**.

Validation II – Two-nucleon separation energies and α -decay energies

Experimental data from Wang et al., CPC 45, 030003 (2021)



Root-mean-square deviations from experiment (MeV)

Model	S_{2n}	S_{2p}	Q_α
TRHB	0.219	0.364	0.368
TRHB+5DCH	0.178	0.321	0.322

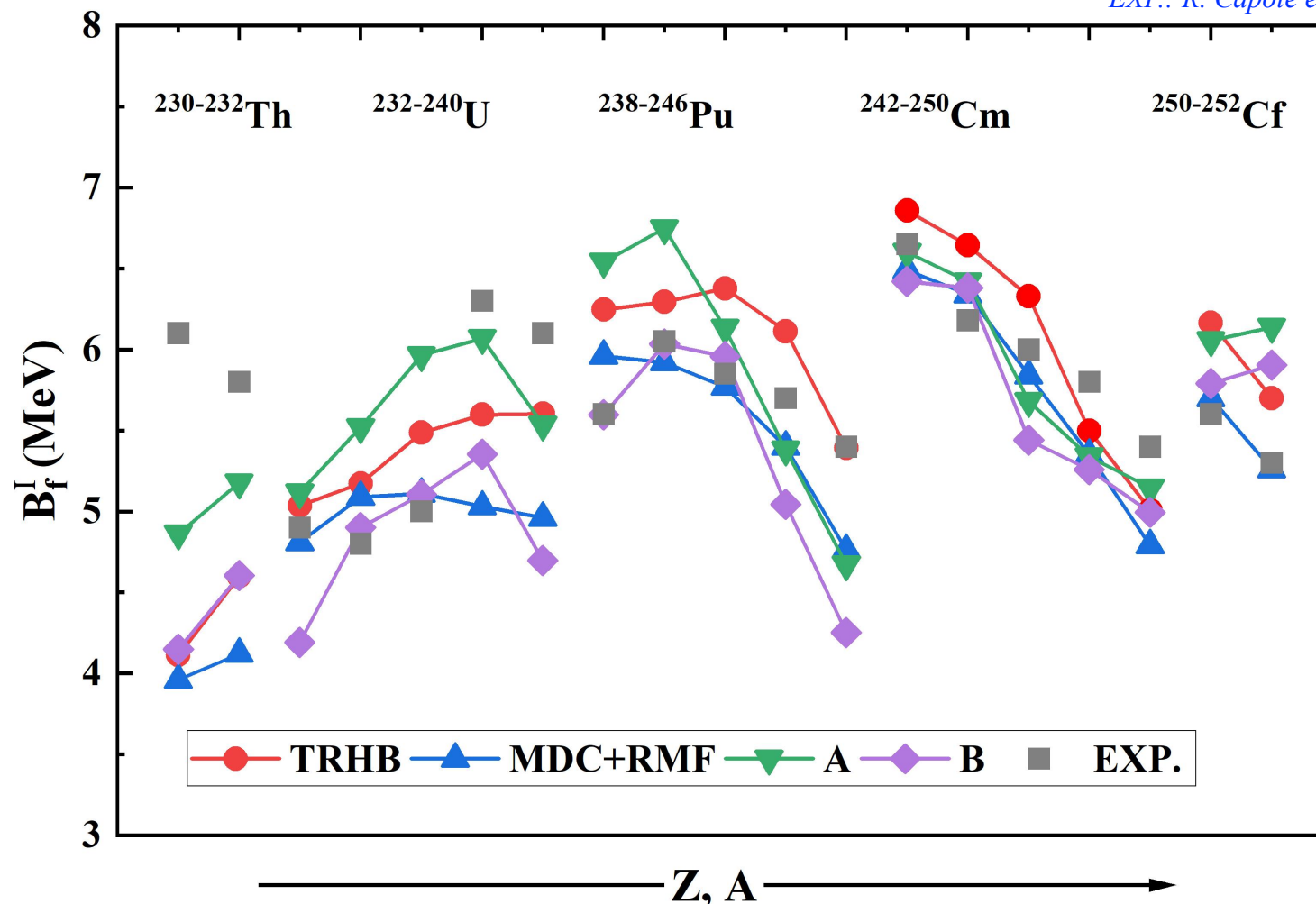
Validation III – fission barriers of actinide nuclei

TRHB: X. Q. Yang, R. Y. Hu, et al., *Phys. Lett. B* 876, 140382 (2026).

MDC+RMF: B.-N. Lu et al., *Phys. Rev. C* 89, 014323 (2014).

A/B: J.-P. Delaroche et al., *Nucl. Phys. A* 771, 103 (2006).

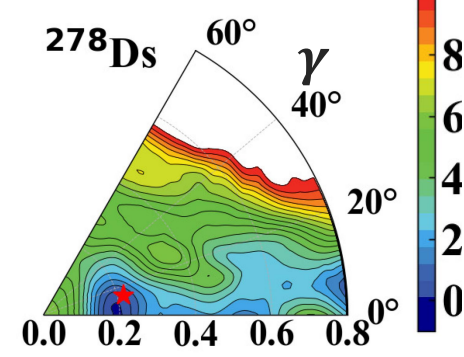
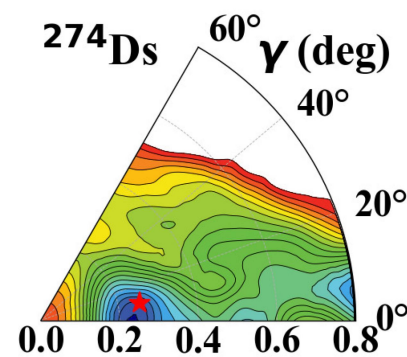
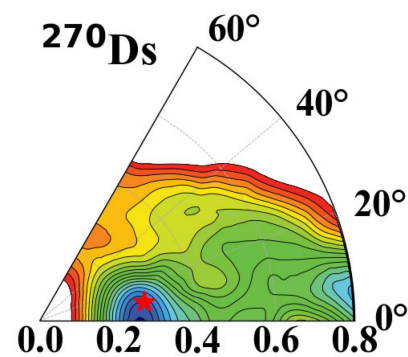
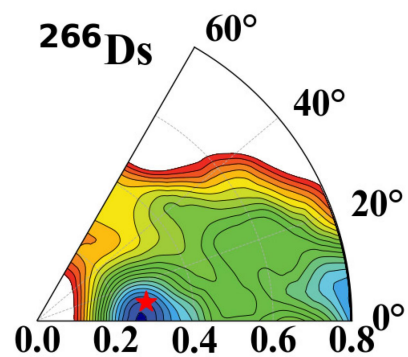
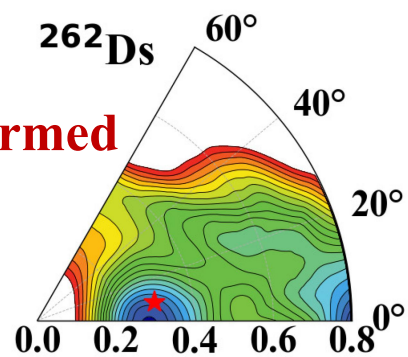
EXP.: R. Capote et al., *Nucl. Data Sheets* 110, 3107 (2009).



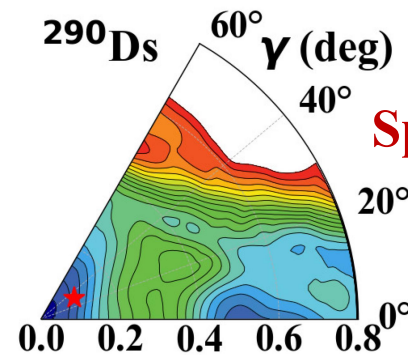
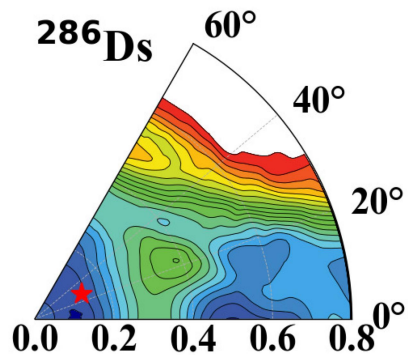
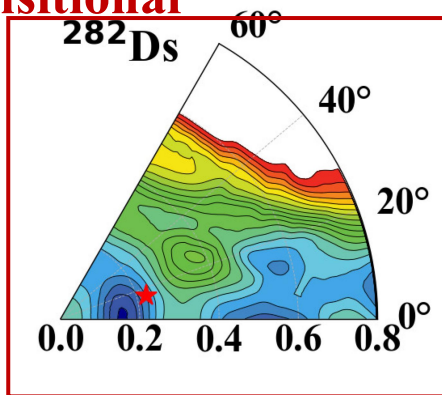
➤ In agreement with empirical values and overall deviation is less than 1 MeV.

Shape Evolution in the Ds ($Z=110$) Isotopic Chain

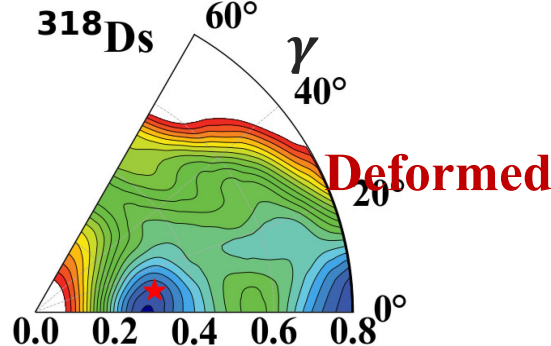
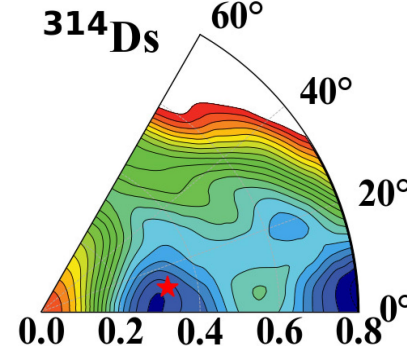
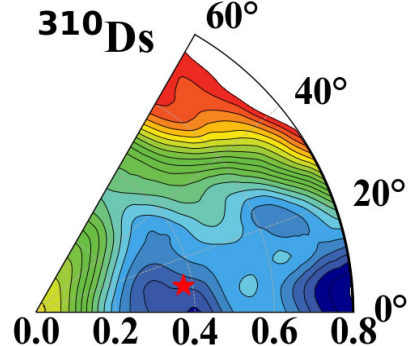
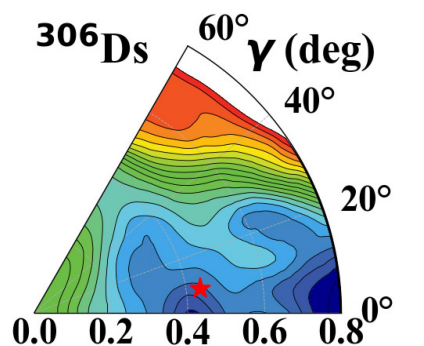
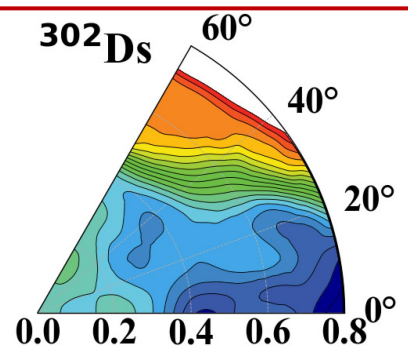
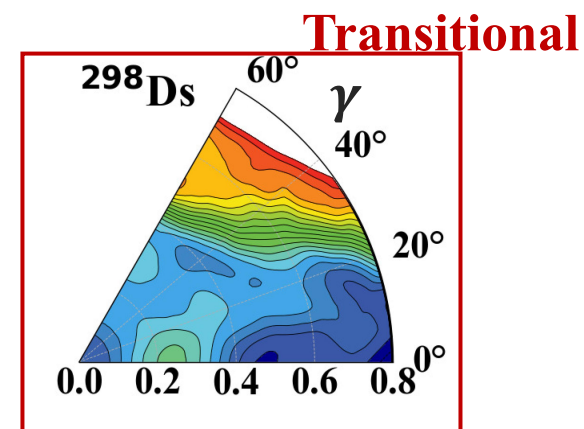
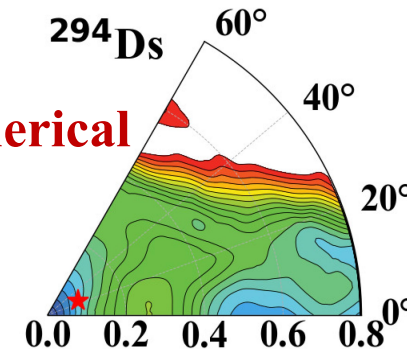
Deformed



Transitional



Spherical

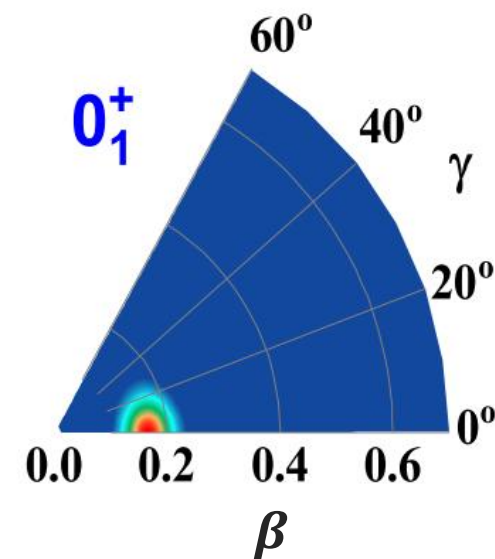
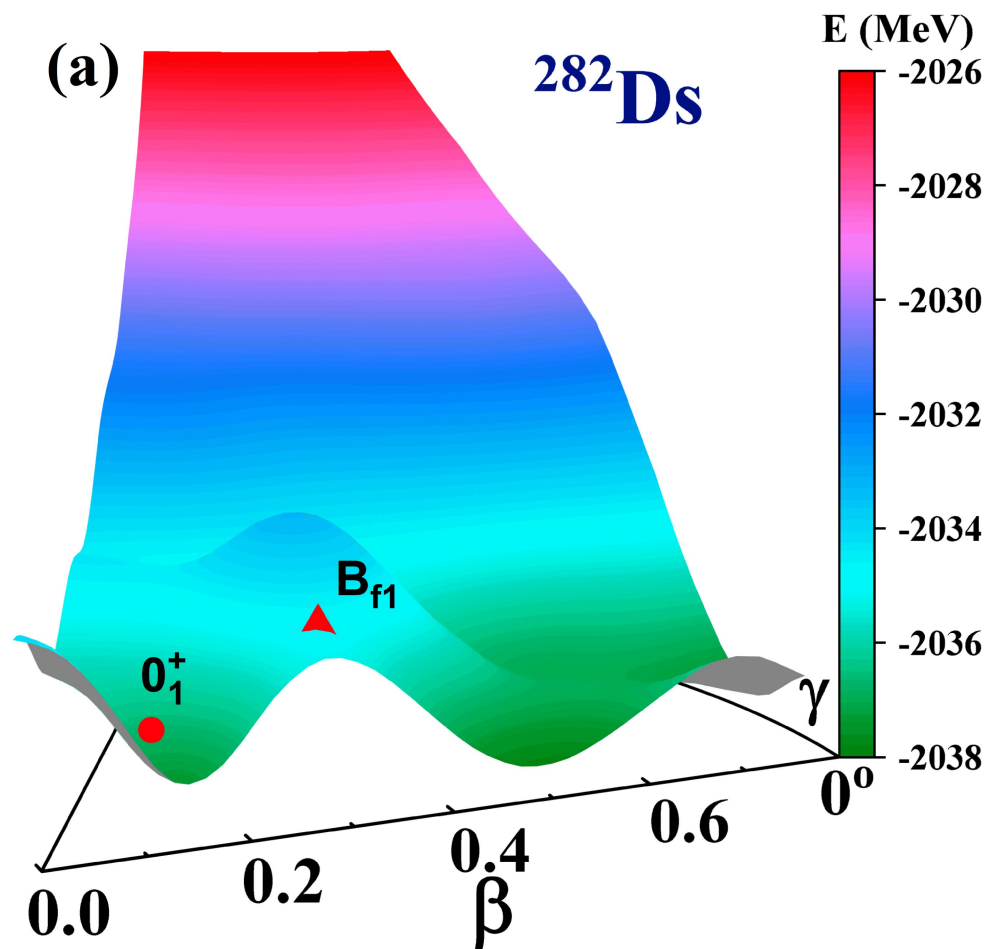


Deformed

 β β β β β

Potential energy surface and 0^+ state in ^{282}Ds

X. Q. Yang, R. Y. Hu, et al., Phys. Lett. B 876, 140382 (2026).



□ “Bound” 0_1^+ state

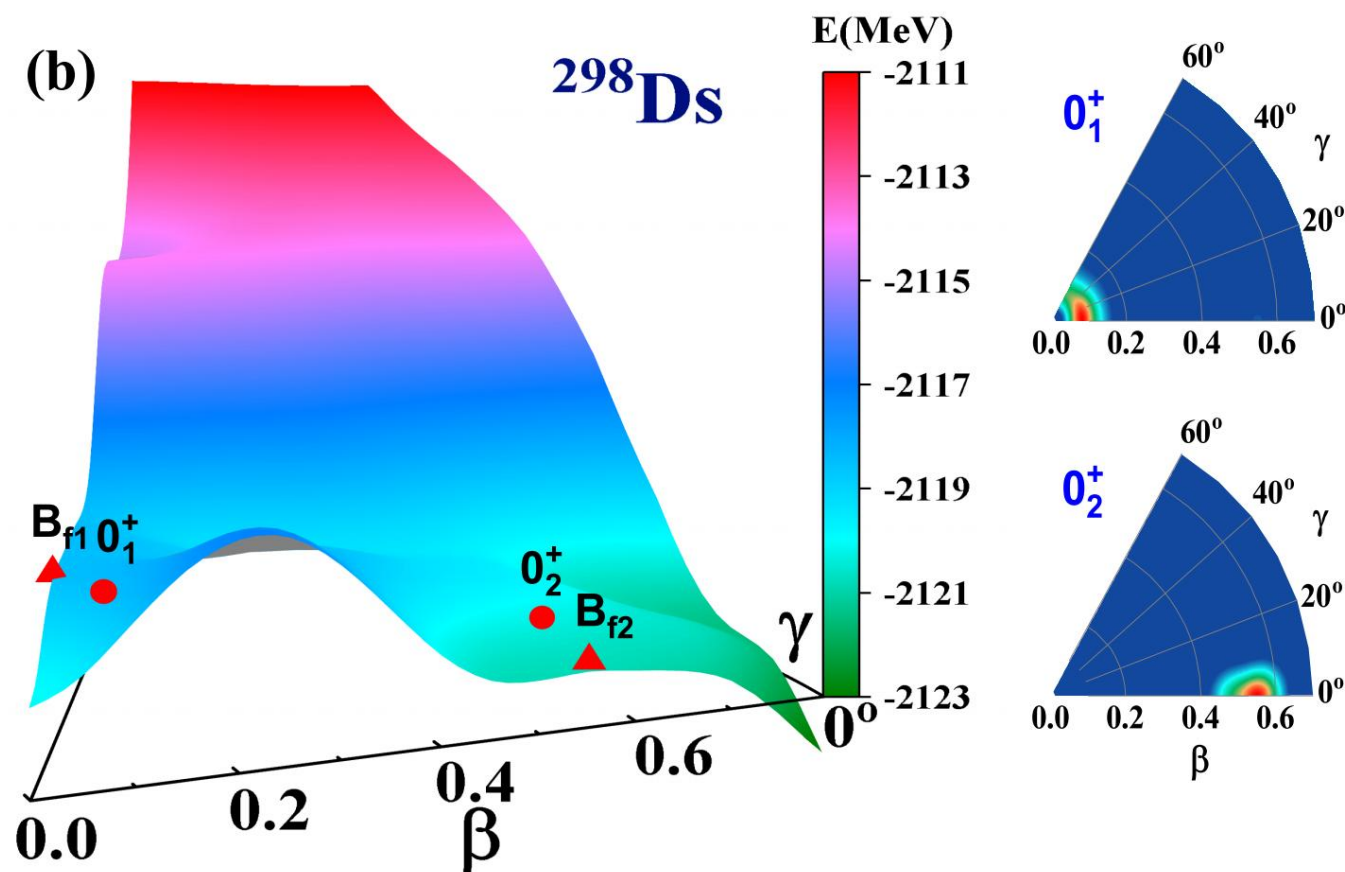
✓ $E(0_1^+) = -2036.42 \text{ MeV}$

✓ $B_{f1} = -2034.94 \text{ MeV}$

➤ In ^{282}Ds , a 0^+ state bound by the fission saddle exists at the minimum

Potential energy surface and 0^+ state in ^{298}Ds

X. Q. Yang, R. Y. Hu, et al., Phys. Lett. B 876, 140382 (2026).



□ “Unbound” 0_1^+ state

✓ $E(0_1^+) = -2118.26 \text{ MeV}$

✓ $B_{f1} = -2118.42 \text{ MeV}$

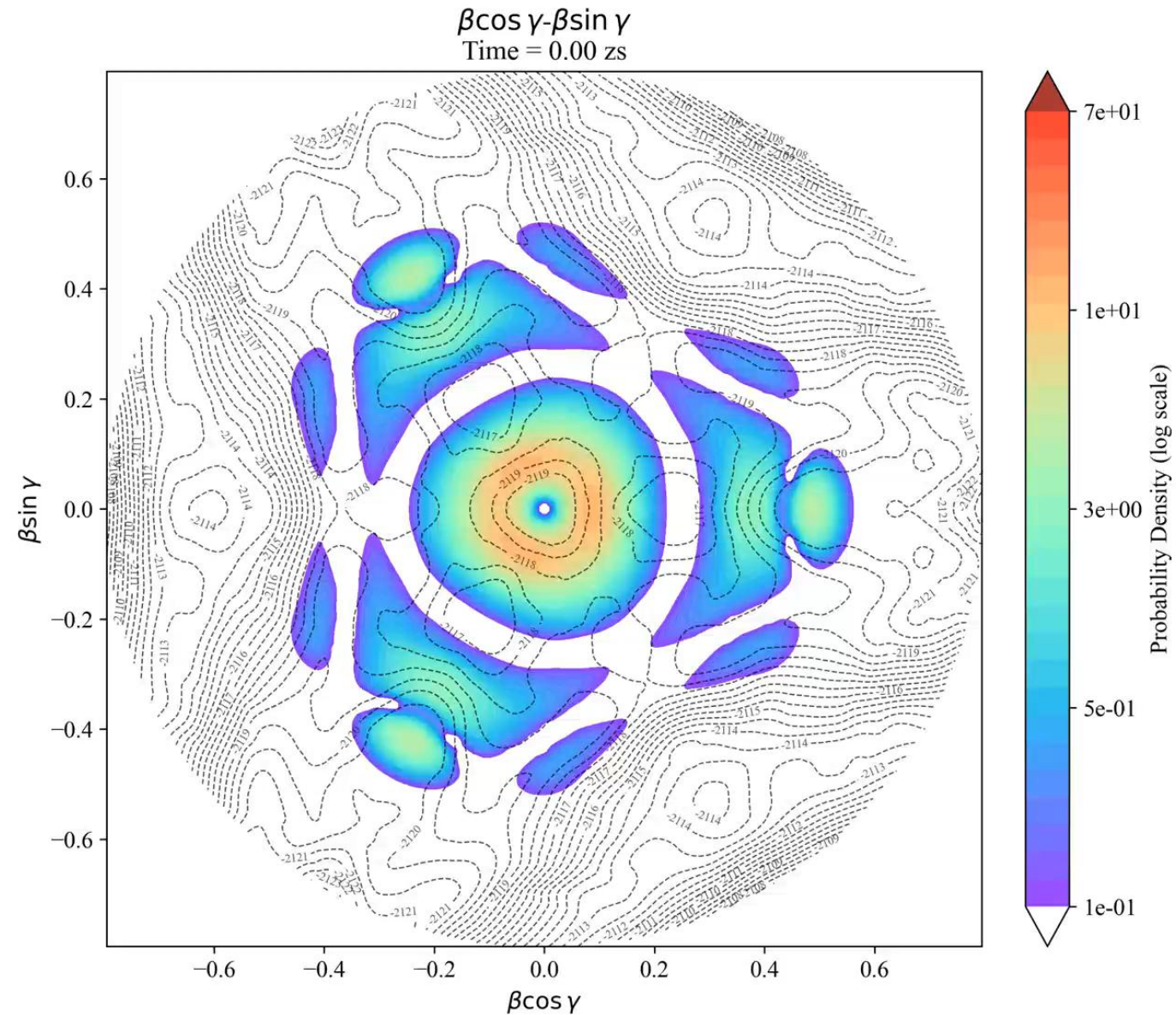
□ “Unbound” 0_2^+ state

✓ $E(0_2^+) = -2120.06 \text{ MeV}$

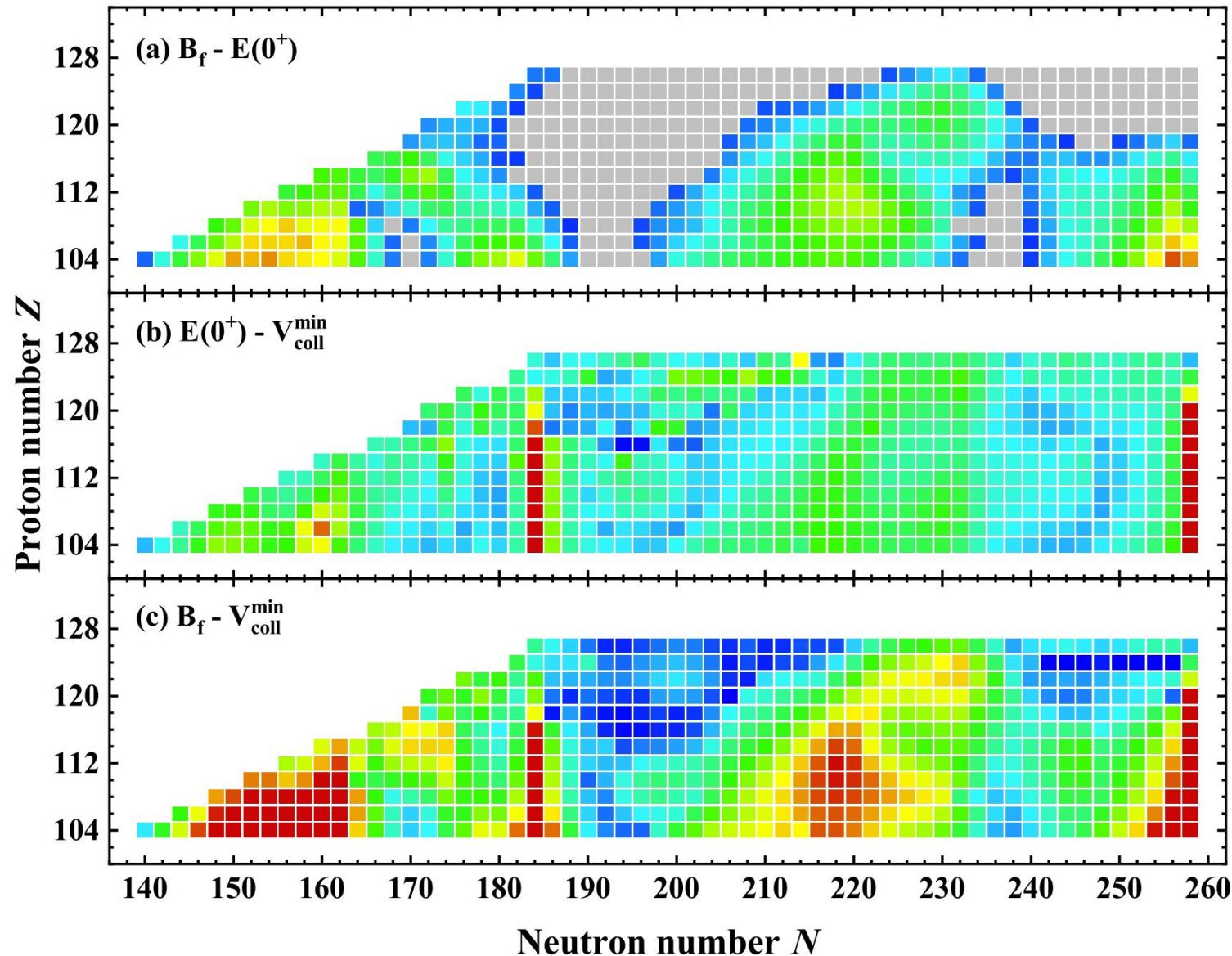
✓ $B_{f2} = -2120.80 \text{ MeV}$

- Both candidate 0^+ states in ^{298}Ds lie above the adjacent fission saddles. Even if formed in a fusion reaction, the nucleus would undergo rapid fission.

Dynamical evolution of the “ 0_1^+ ” state in ^{298}Ds



0^+ states and fission barriers

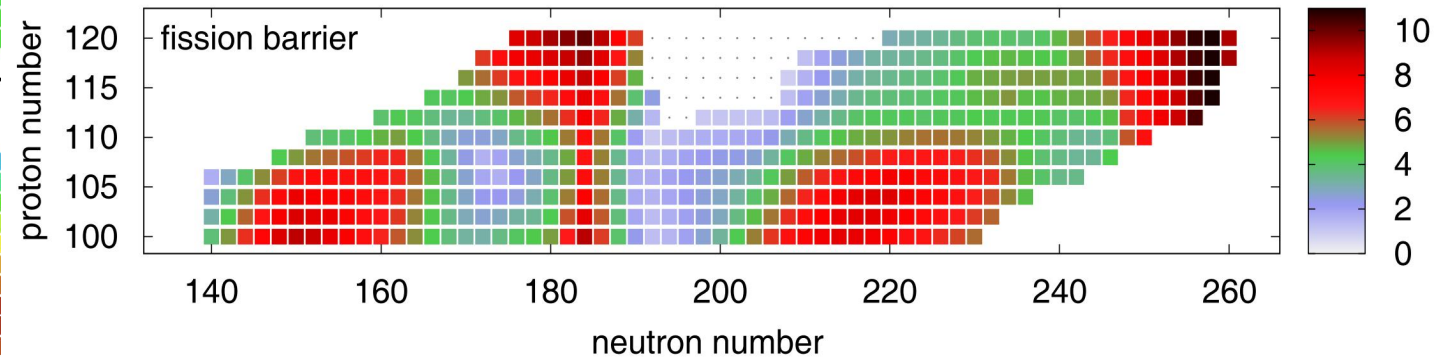
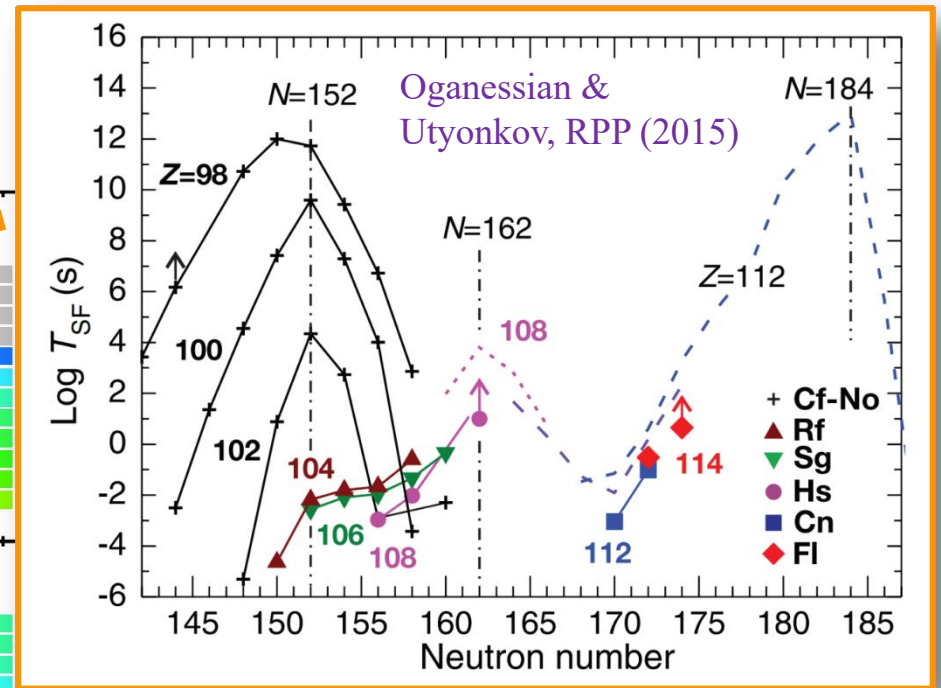
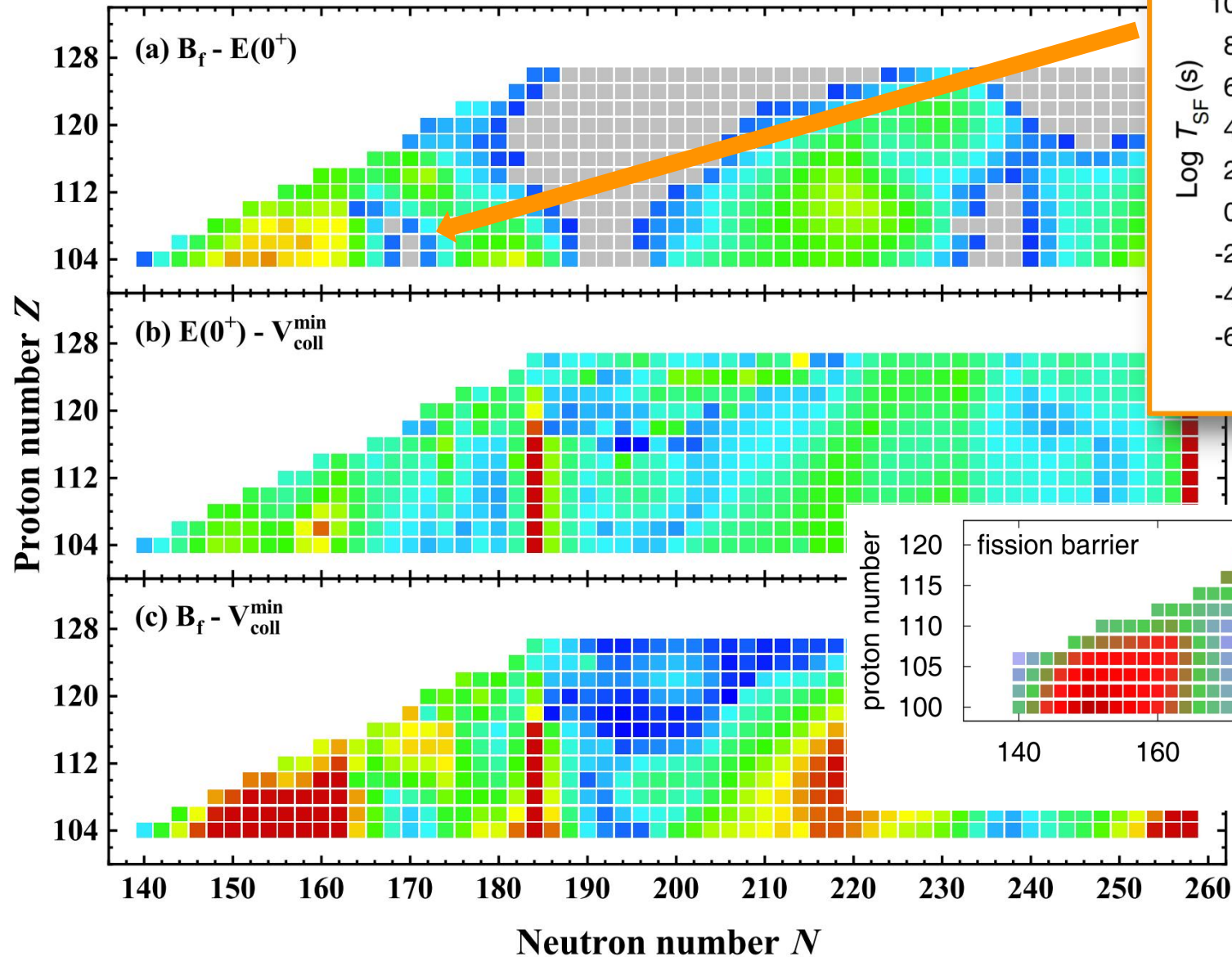


➤ “Unbound” appear in transitional regions near $N = 184/240$, as well as around $N \approx 170$ and $N \approx 236$.

➤ Collective zero-point energies are significant near magic numbers and in well-deformed regions, but smaller in transitional regions (soft PES).

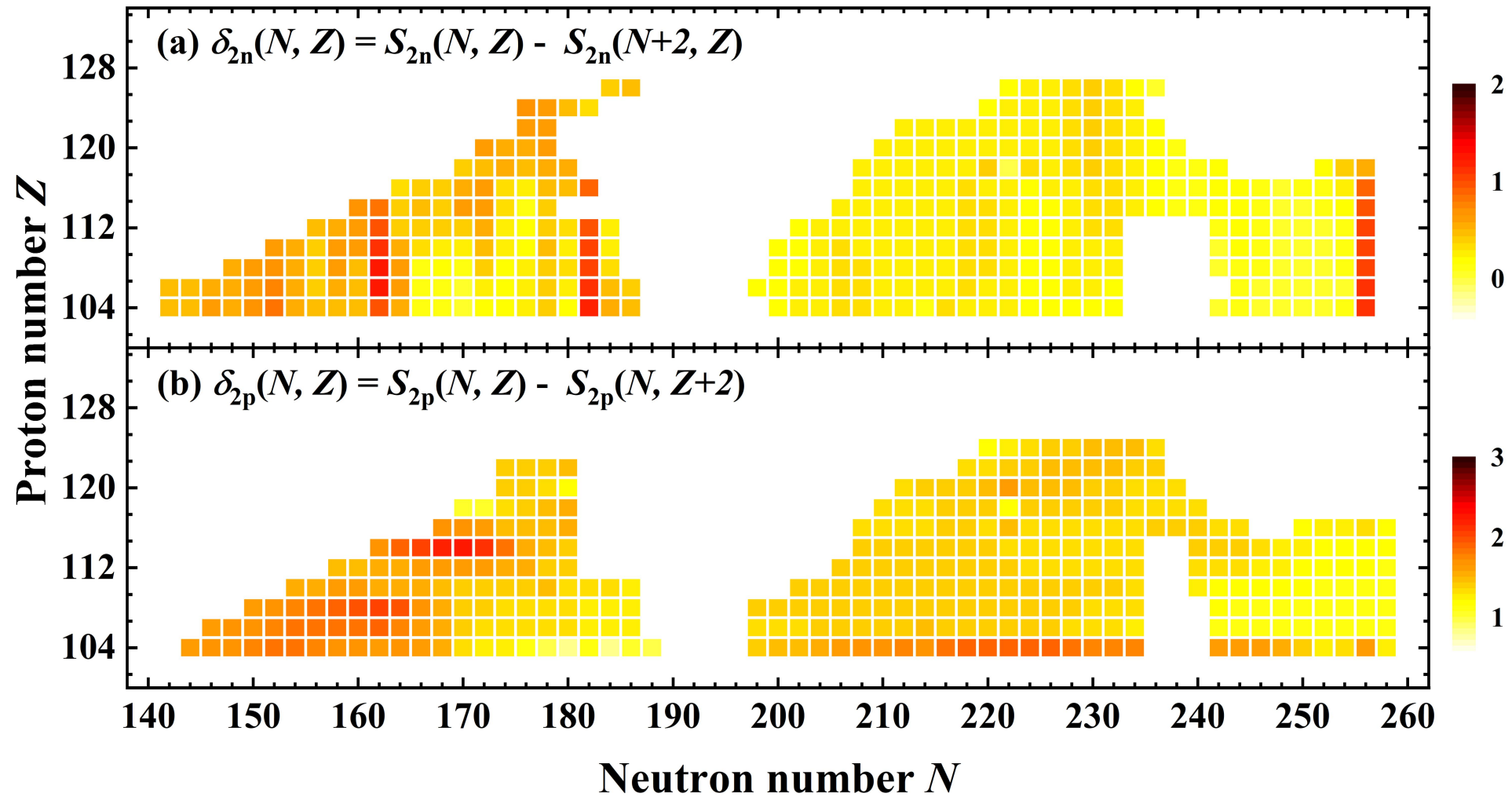
➤ Fission barriers are high near magic numbers and in deformed regions, but very low in transitional regions.

0⁺ states and fission barriers



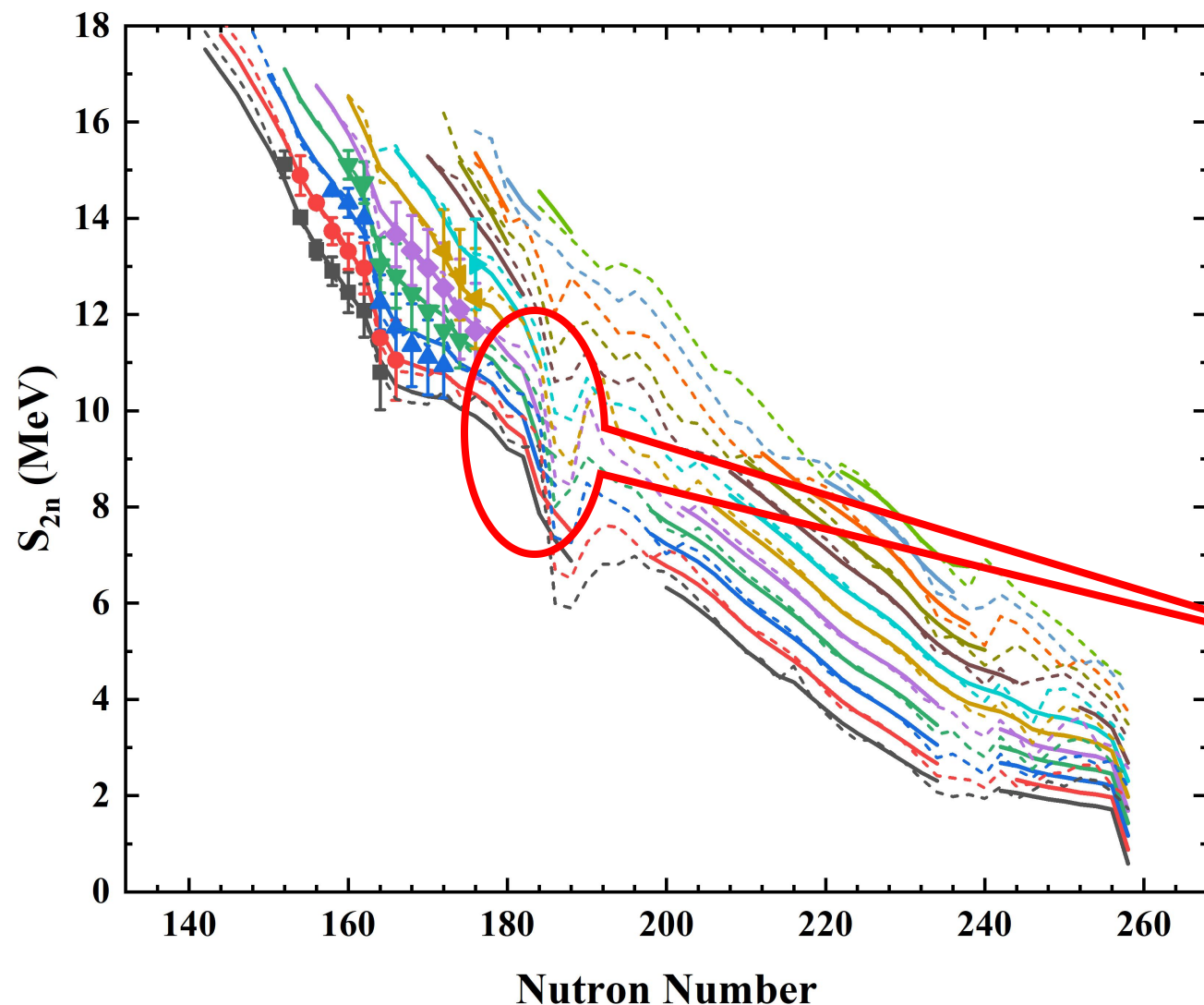
SHF results (SV-min): P-G. Reinhard, Eur. Phys. J. A (2018) 54: 13

Two-Nucleon Shell Gaps



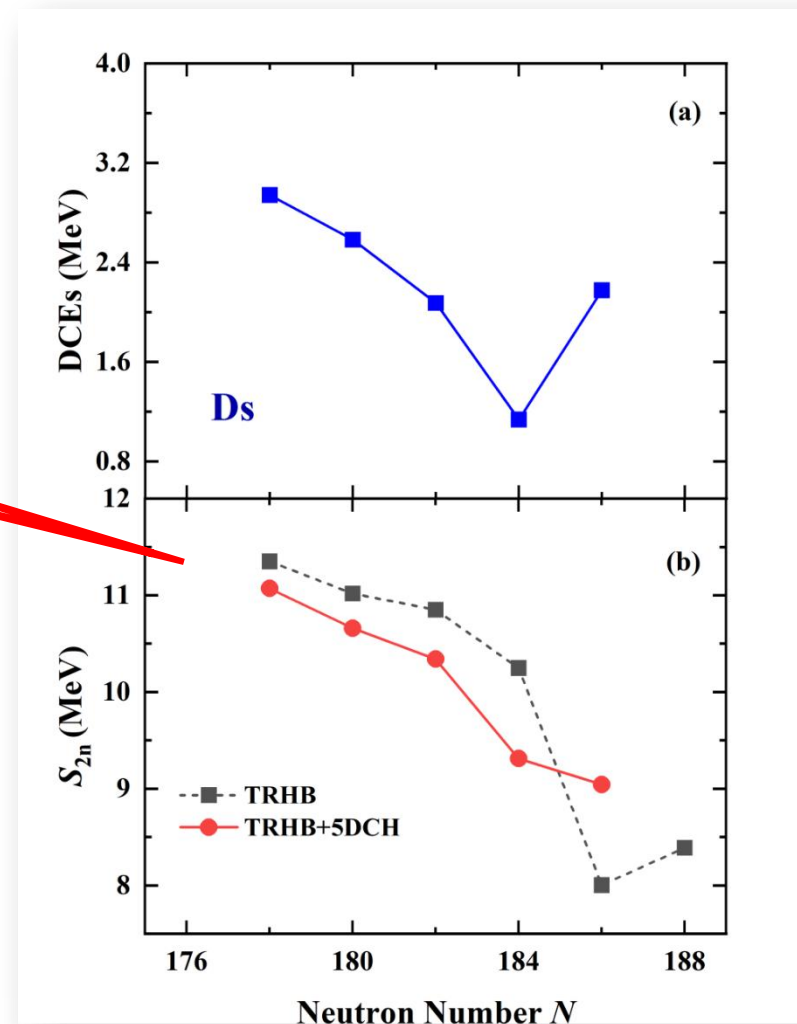
- Two-neutron shell gaps appear at $N = 162, 182, \text{ and } 256$;
- Two-proton shell gaps appear for neutron-deficient $Z = 114$ and neutron-rich $Z = 104$.

Impact of Collective Dynamical Correlations on S_{2n} and Shell Gaps



Experimental data from Wang et al., CPC 45, 030003 (2021)

- S_{2n} exhibits discontinuities (1–2 MeV) at $N = 162, 182,$ and 256 .



Outline

1. Introduction

2. Microscopic Collective Hamiltonian Model

3. Structure and Stability of Superheavy Nuclei

4. Summary and Outlook



Summary and Outlook

X. Q. Yang, R. Y. Hu, et al., Phys. Lett. B 876, 140382 (2026).

- **Systematic PC-PK1 collective-Hamiltonian study of even-even superheavy nuclei**
 - Masses, two-nucleon separation energies and α -decay energies agree with data;
 - Shallow transitional wells may not support a 0^+ states below the adjacent fission saddle reveals the phenomenon of "unbound" 0^+ states;
 - Shape evolution: Well-deformed prolate ($N \sim 150, 210$) $\rightarrow$ medium-deformed γ -soft ($N \sim 176, 246$) $\rightarrow$ spherical ($N = 184, 258$);
 - Shell closures identified at $Z = 114$; $N = 162, 182, 256$.
- Investigate octupole effects on fission barriers and “unbound” 0^+ states using the microscopic TQOCH.
- Odd-nucleon extension of the collective Hamiltonian.

The background of the slide features a large, faint watermark of the Southwest University logo. The logo is circular, with the university's name in Chinese characters '西南大學' at the top and 'SOUTHWEST UNIVERSITY' at the bottom. In the center is a stylized emblem. The entire slide has a solid blue background.

Thank you for your attention!

Collaborators: Xiaoqin Yang (SWU, IMP), Ruonan Mao (SWU), Jian Xiang (CQNU), Zhipan Li (SWU)