

Microscopic description of spontaneous fission for Fm isotopes

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- Collective inertia in fission by DFT
- QRPA vs. cranking for collective inertia
- Fission half-life in Fm isotopes
- Discussion on pairing

Washiyama, Hinohara, Nakatsukasa, PRC103, 014306 (2021)

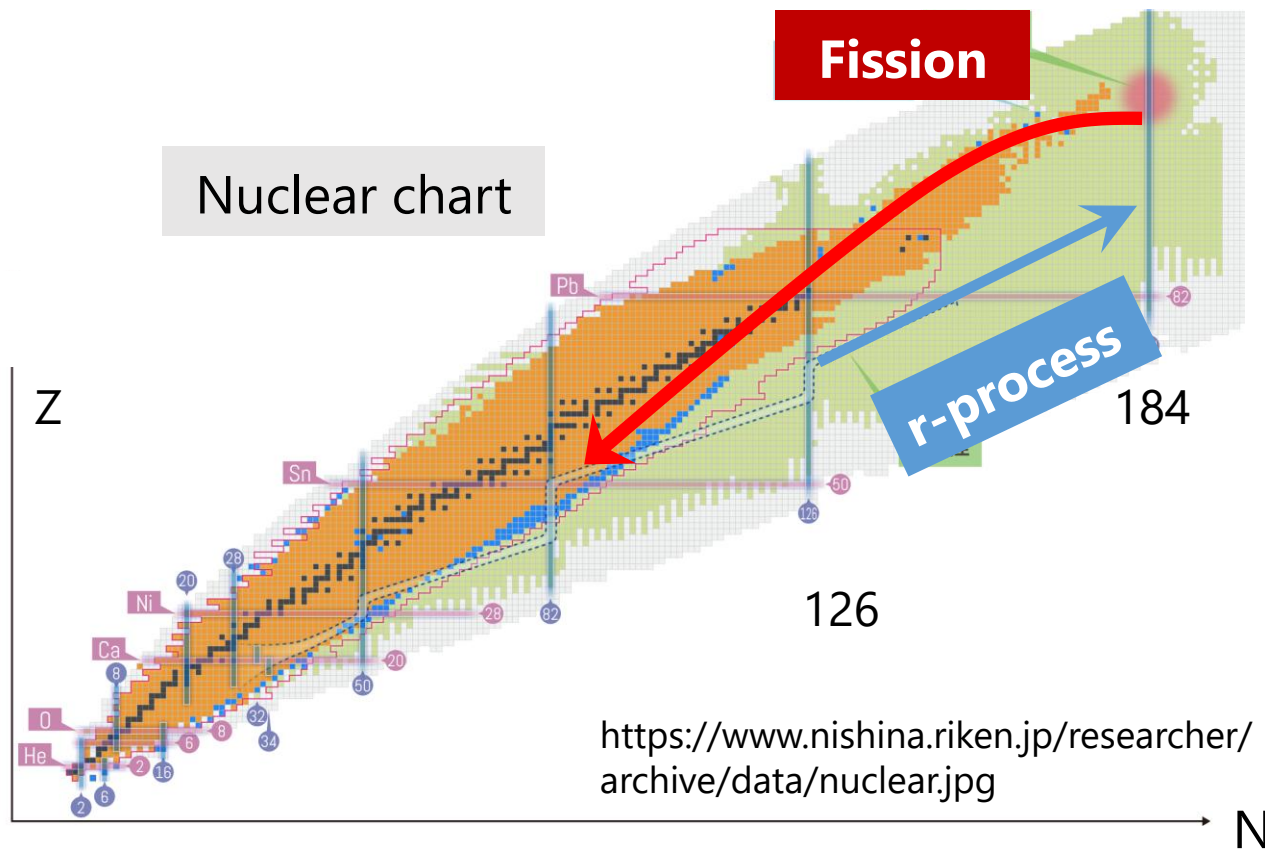
Washiyama, Web of Conf. 306, 01026 (2024)



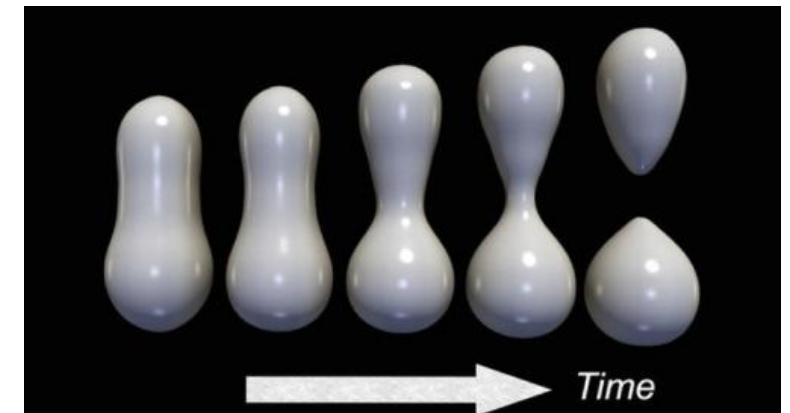
Introduction: Nuclear fission

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- Decay mode in heavy nuclei
- End point of r-process



- Large amplitude collective motion
- Many-body quantum tunneling



Courtesy of G. Scamps

WKB approximation

J. Sadhukhan et al, PRC88, 064314 (2013), etc.

Fission half-life

$$T_{1/2} = \ln 2 / (nP) \quad P = \frac{1}{1 + \exp(2S)}$$

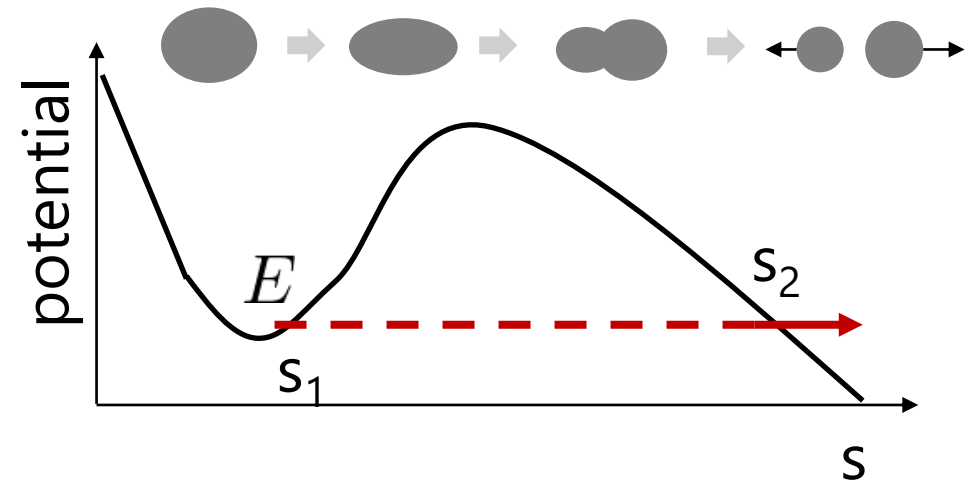
Action

$$S = \int_{s_1}^{s_2} ds \sqrt{2M(s)(V(s) - E)}/\hbar$$

s Collective coordinate (Q_{20} etc.)

$V(s)$ Fission barrier: Constrained DFT on s

$M(s)$ **Collective inertia: DFT + Cranking formula**



Former work: Cranking formula

Neglect residual effects

Scaling factor $D = (1 + \alpha) D^{\text{crank}}$

Baran et al., PRC84, 054321 (2011)

Sadhukhan et al, PRC88, 064314 (2013)

Rodriguez-Guzman, Robledo, PRC89, 054310

Giuliani and Robledo, PLB 787, 134 (2018)

Our work: **Local QRPA**

Include residual effects

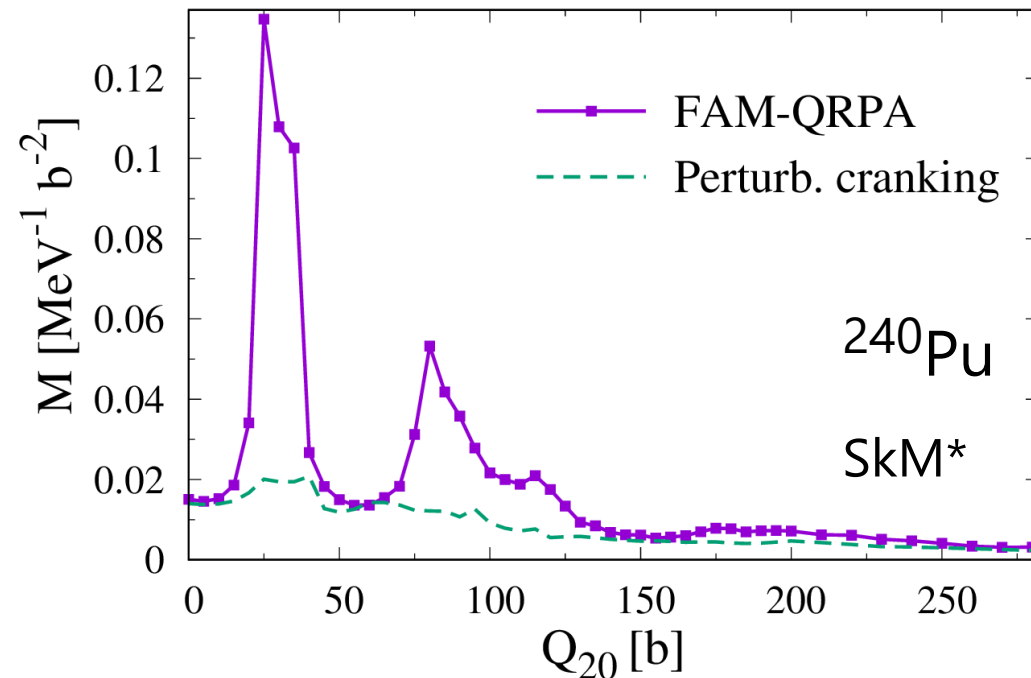
Washiyama, Hinohara, Nakatsukasa,
PRC103, 014306 (2021)

Hinohara et al., PRC82 (2010) 064313

Sato, Hinohara, NPA849(2011)53

Yoshida, Hinohara, PRC83(2011)061302

Comparison (QRPA/Cranking)



- Strong enhancement
- Strong Q_{20} dependence

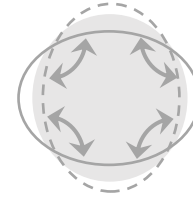
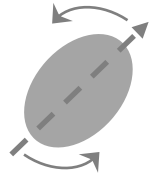
Washiyama, Hinohara, Nakatsukasa, PRC103, 014306 (2021)

Inertial functions in five-dimensional collective Hamiltonian

$$\mathcal{H} = T_{\text{vib}} + T_{\text{rot}} + V(\beta, \gamma)$$

$$T_{\text{vib}} = \frac{1}{2} D_{\beta\beta}(\beta, \gamma) \dot{\beta}^2 + D_{\beta\gamma}(\beta, \gamma) \dot{\beta} \dot{\gamma} + \frac{1}{2} D_{\gamma\gamma}(\beta, \gamma) \dot{\gamma}^2$$

$$T_{\text{rot}} = \frac{1}{2} \sum_{k=1}^3 \mathcal{J}_k(\beta, \gamma) \omega_k^2$$



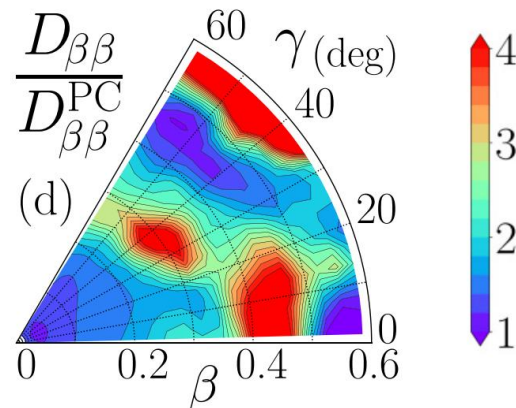
Bohr & Mottelson, Nuclear Structure Vol. II, Chap. 6B
Kumar & Baranger, NPA92, 608 (1967)

Prochniak et al., NPA730 (2004) 59
Niksic et al., PRC79 (2009) 034303
Delaroche et al., PRC81 (2010) 014303, etc.

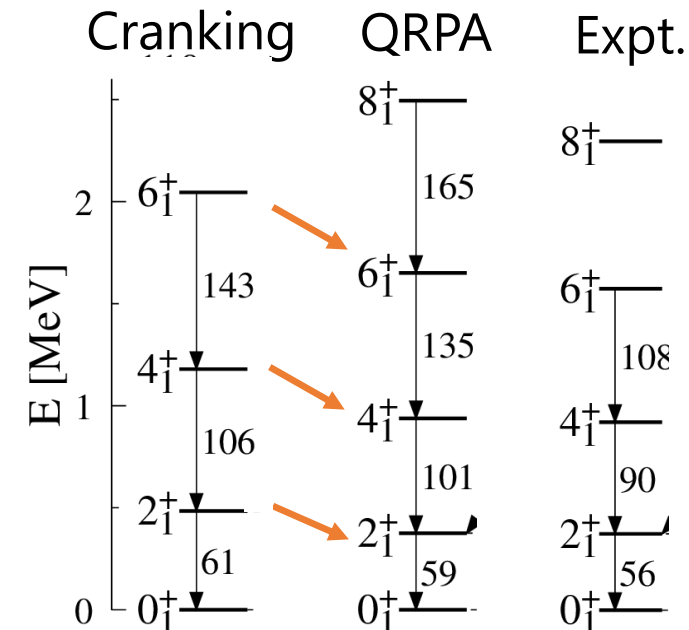
QRPA inertial functions for 5DCH

Washiyama, Hinohara, Nakatsukasa,
Phys. Rev. C 109, L051301(2024)

Enhancement of
QRPA/cranking



Improved spectra in ^{110}Pd



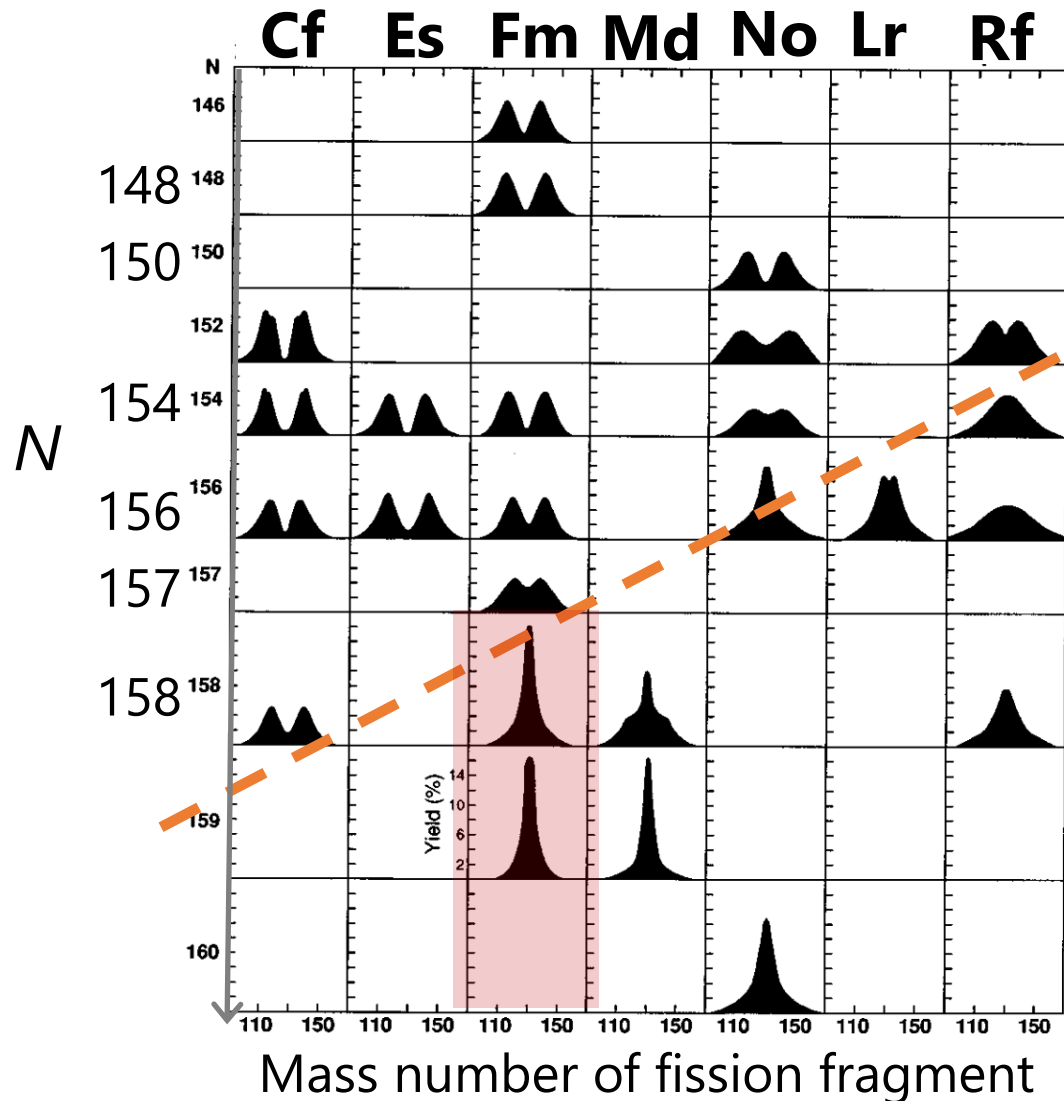
3D Skyrme DFT
SkM*

B(E2) [W.u.]

Objective: Fission in Fm isotopes

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Mass distribution of fission fragments



Mass symmetric fission is dominant in Fm isotopes, $N \geq 158$



Setting $Q_{30} = 0$ may not be a bad approximation

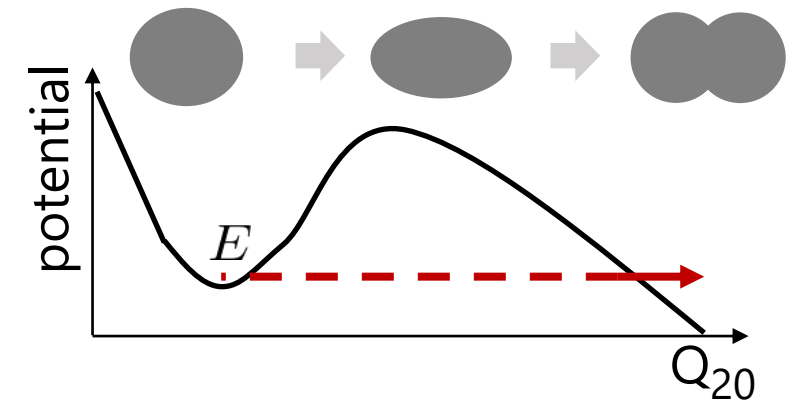
M.R. Lane et al., PRC53, 2893 (1996)

$$S = \int_{s_1}^{s_2} ds \sqrt{2M(s)(V(s) - E)/\hbar}$$

$V(s)$: Constrained Skyrme HFB along the fission path

Constrained on Q_{20}

Fix $Q_{30} = 0 \rightarrow$ Symmetric fission path only



$M(s)$: Local QRPA on constrained HFB states with the finite amplitude method

$$\delta\langle\phi(s)|[\hat{H}_M(s), \hat{Q}^i(s)] - \frac{1}{i}\hat{P}^i(s)|\phi(s)\rangle = 0$$

$$\delta\langle\phi(s)|[\hat{H}_M(s), \frac{1}{i}\hat{P}^i(s)] - C_i(s)\hat{Q}^i(s)|\phi(s)\rangle = 0$$

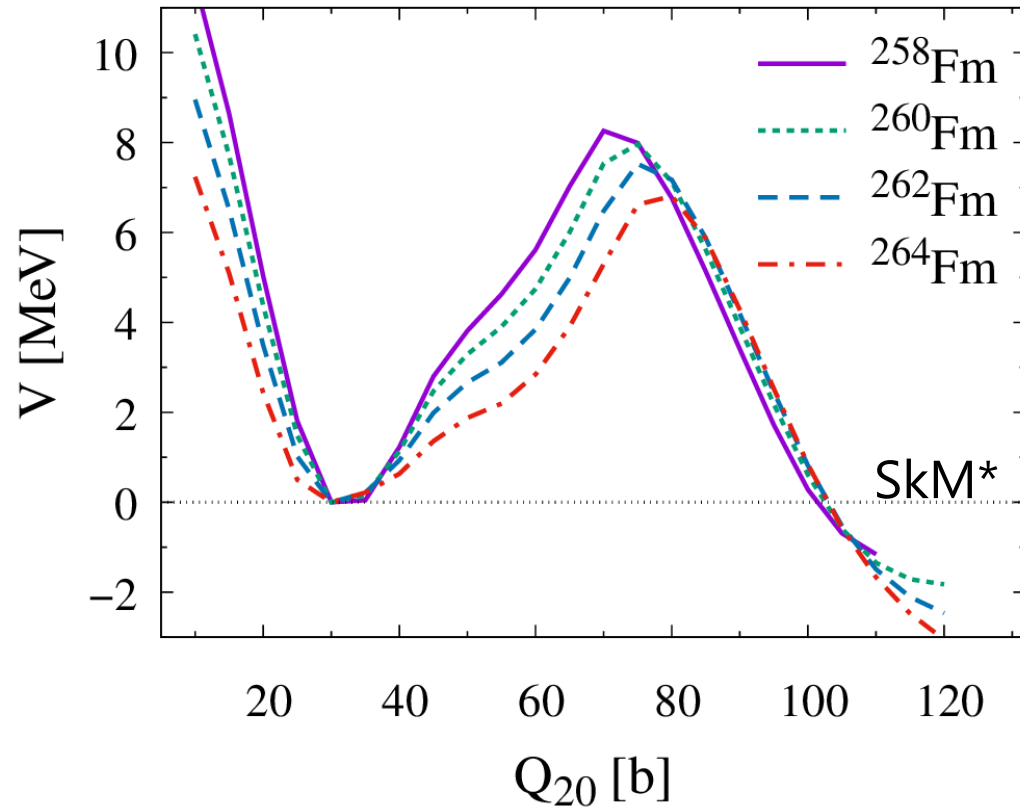
➡ Select the most collective mode

Computation time: 60000 - 70000 hours (for 30 states)

Washiyama, Hinojara, Nakatsukasa, PRC103, 014306 (2021)

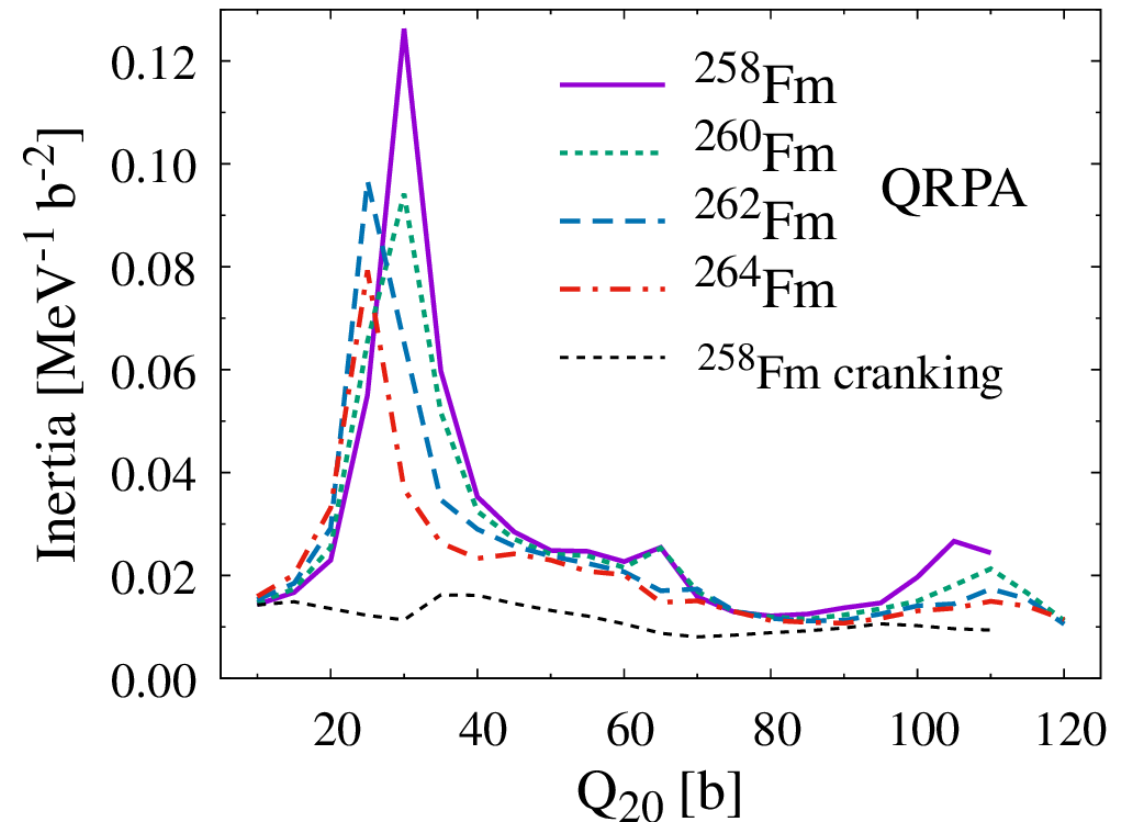
$E = 0$ MeV

Fission barrier



$^{258}\text{Fm} > ^{260}\text{Fm} > ^{262}\text{Fm} > ^{264}\text{Fm}$

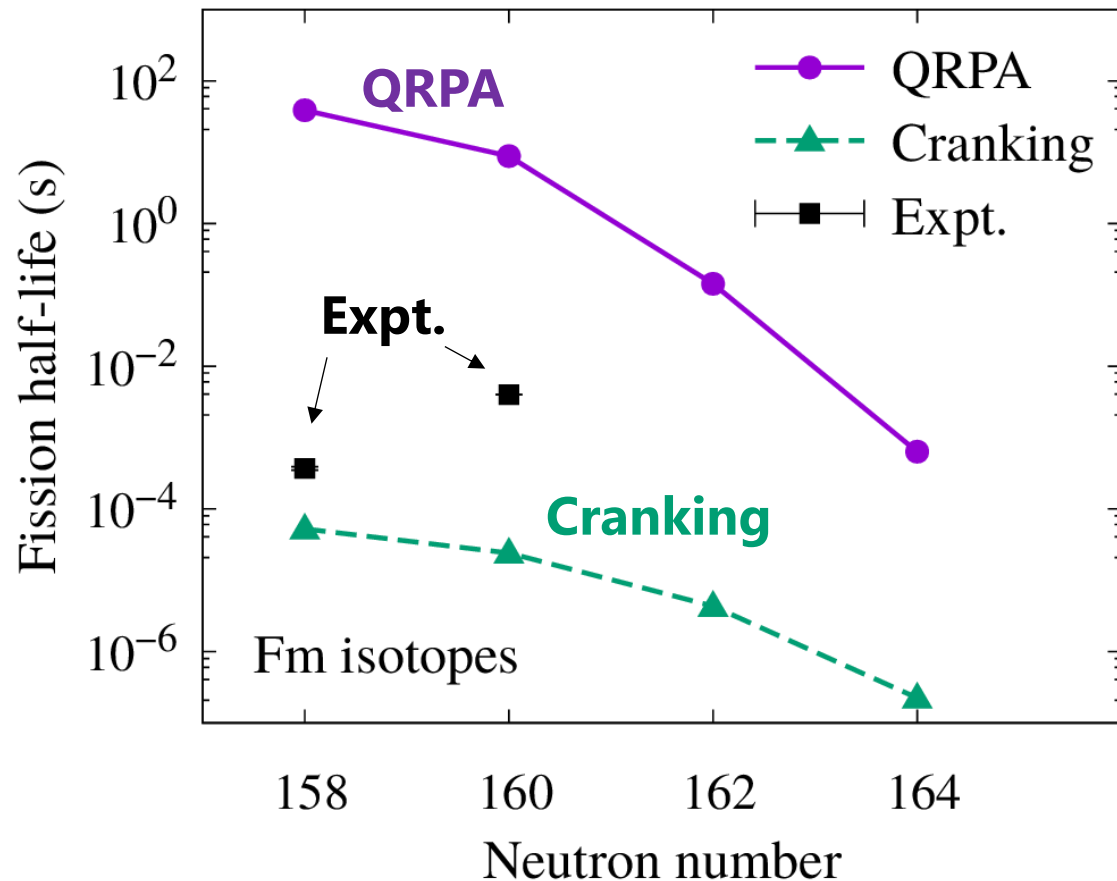
Inertia



QRPA > Cranking

SkM* + volume pairing
 $25 \times 25 \times 35 \text{ fm}^3$, $dx=1.0 \text{ fm}$

Fission Half-life



$$T_{1/2} = \ln 2 / (nP)$$

$$n = 10^{20.38} \text{ s}^{-1} \quad P = \frac{1}{1 + \exp(2S)}$$

A. Baran, Phys. Lett. B 76, 8 (1978)

Sadhukhan et al, PRC88, 064314 (2013), etc.

QRPA vs. cranking inertia

5—6 orders of difference

Impact on half-life

Overestimation with QRPA inertia

Data from N. E. Holden and D. C. Hoffman,
Pure Appl. Chem. 72, 1525 (2000)

Washiyama, EPJ Web of Conf. 306, 01026 (2024)

Overestimation of the calculated half-lives in QRPA may be improved by including additional collective coordinates

→ Decrease the action $S = \int_{s_1}^{s_2} ds \sqrt{2M(s)(V(s) - E)}/\hbar$

Additional collective coordinates

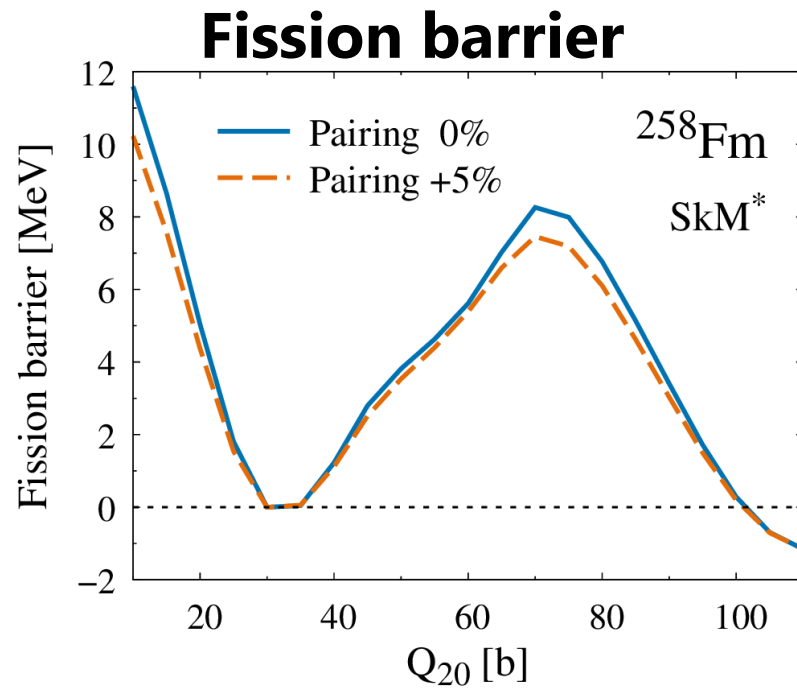
- Octupole moment, nonaxial (triaxial) quadrupole moment

- **Pairing**

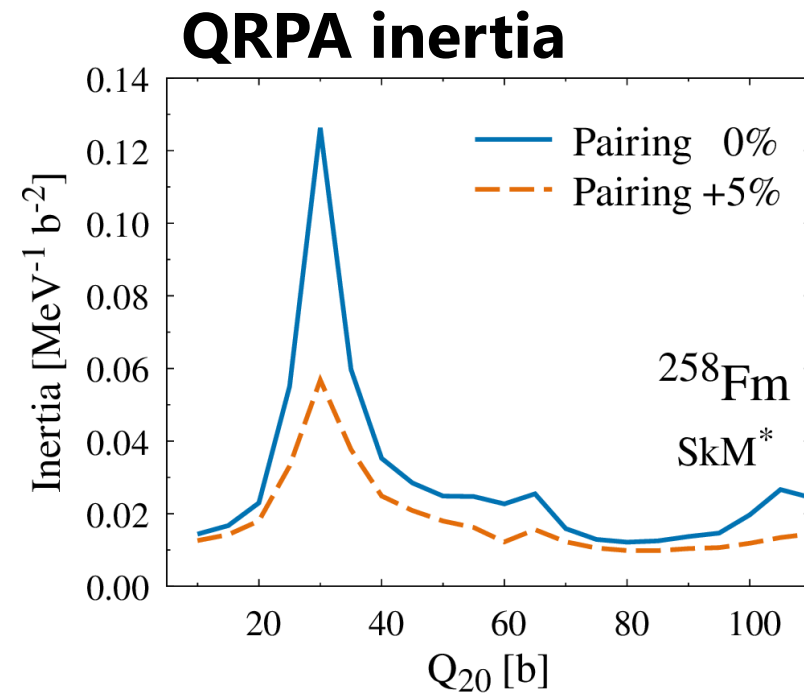
→ **Induce a large variation in the inertia**

“Pairing-induced speedup of nuclear spontaneous fission”

J. Sadhukhan et al, PRC90, 061304 (2014)



10% change
at barrier



20 – 50%
change

Action S

25.6 (Pairing 0%)

20.6 (Pairing 5%)



Half-life

4.5×10^1 s (Pairing 0%)

2.1×10^{-3} s (Pairing 5%)

$0.37(2) \times 10^{-3}$ s (data)

+5% pairing



4 order speedup
in half-life

Spontaneous fission by DFT + WKB approximation

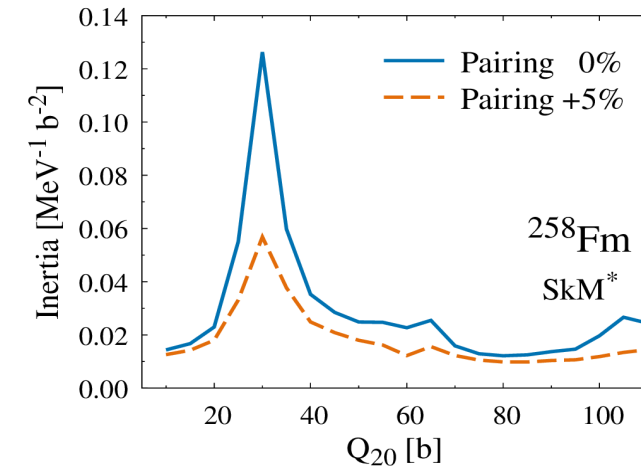
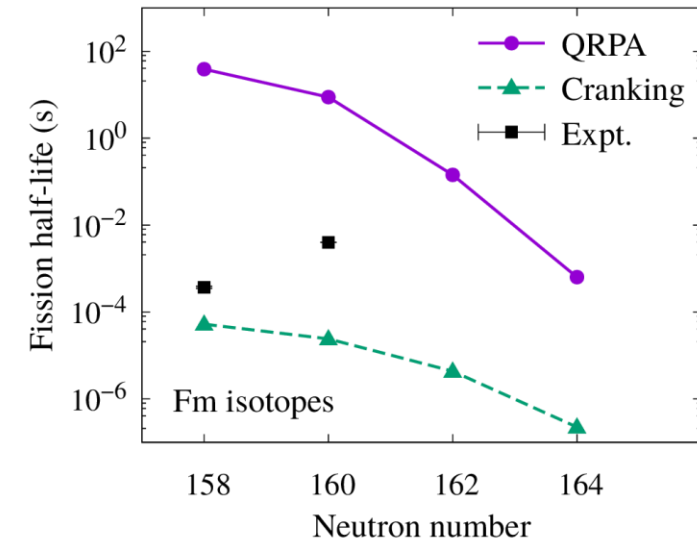
Collective inertia improved by local QRPA

Impact of collective inertia on fission half-life

Future work

Multiple collective coordinates

- Octupole and triaxial quadrupole moment
- Pairing as a collective coordinate



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