



Stochastic Bogoliubov-de Gennes Approach to Nonequilibrium Fermionic Superfluidity

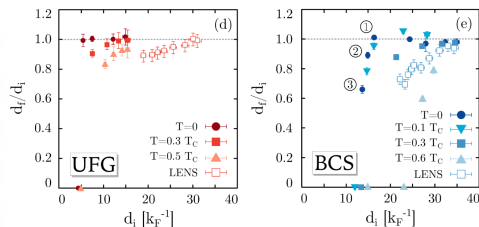
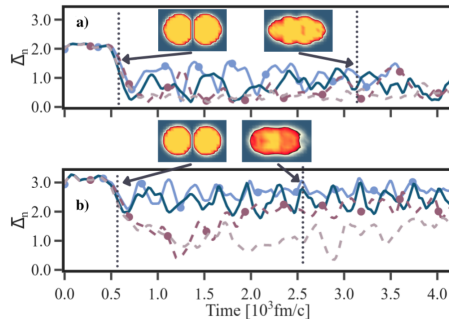
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Introduction

A. Barresi et al., Phys. Rev. Lett. 130, 043001, 2023

P. Magierski et al., Phys. Rev. C 105, 064602, 2022

- Mean-Field theories (MF) $\rightarrow$ particle interacts with average field from other particles
 - Dissipative processes are underestimated
- $\Downarrow$
- Thermalization times are overestimated and collective modes are stable
 - Fluctuations acts as two-body correlations, extending MF theories



- Stochastic initial ensemble

$$\rho^{(n)} = \overline{\rho^{(n)}} + \delta\rho^{(n)}$$

evolved through deterministic evolution:

$$i\hbar \frac{\partial}{\partial t} \rho^{(n)}(t) = [h(\rho^{(n)}), \rho^{(n)}(t)]$$

- Add fluctuating term in the time-evolution equation

$$\begin{aligned}
 i\hbar \dot{\phi}_k(\mathbf{r}, t) = & \underbrace{h[n]\phi_k(\mathbf{r}, t)}_{\text{mean field}} + \underbrace{\hbar\gamma[n]\dot{n}(\mathbf{r}, t)\phi_k(\mathbf{r}, t)}_{\text{friction term}} \\
 & - \underbrace{\frac{1}{2}[\mathbf{u}(\mathbf{r}, t) \cdot \hat{\mathbf{p}} + \hat{\mathbf{p}} \cdot \mathbf{u}(\mathbf{r}, t)]\phi_k(\mathbf{r}, t) + u_0(\mathbf{r}, t)\phi_k(\mathbf{r}, t)}_{\text{stochastic terms}}
 \end{aligned}$$

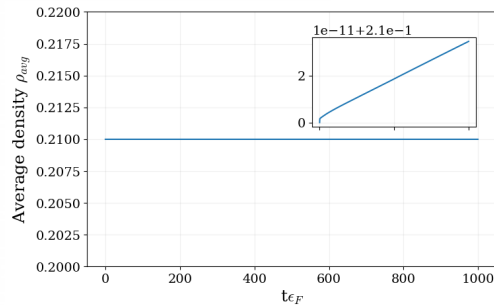
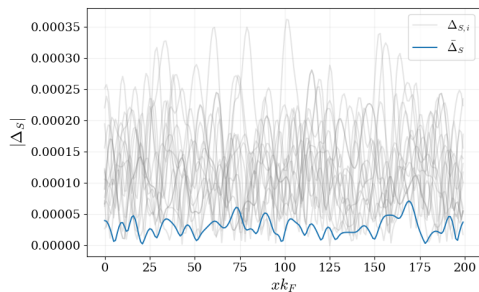
- We are solving the Time-Dependent Bogoliubov-de Gennes equations which have the form:

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_n(\mathbf{r}, t) \\ v_n(\mathbf{r}, t) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r}, t) - \mu & \Delta(\mathbf{r}, t) \\ \Delta(\mathbf{r}, t)^* & -h^*(\mathbf{r}, t) + \mu \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}, t) \\ v_n(\mathbf{r}, t) \end{pmatrix}$$

- We add the stochastic noise in the particle-particle channel, together with the quantum cooling, which will neutralize the heating effect of the stochastic term:

$$\Delta = \Delta_{MF} + \Delta_S + \Delta_{cool}$$

where Δ_{MF} is the mean-field contribution



$$\mathbb{E}[\Delta_0(\mathbf{r}, t)] = 0,$$

$$\mathbb{E}[\Delta_0(\mathbf{r}, t) \Delta_0^*(\mathbf{r}', t')] = D_\Delta C_r(\mathbf{r} - \mathbf{r}') C_t(t - t')$$

$$\Delta_S(\mathbf{r}, t) = \Delta_0(\mathbf{r}, t) - i \frac{\text{Im} \int d^d r' \chi^*(\mathbf{r}', t) \Delta_0(\mathbf{r}', t)}{\int d^d r' |\chi(\mathbf{r}', t)|^2} \chi(\mathbf{r}, t)$$

$\Downarrow$

$$\frac{dN}{dt} \sim \text{Im} \left(\text{Tr}(\Delta_S^* \chi) \right) = 0.$$

J.E. Alba-Arroyo et al., Phys. Rev. C 113, 065805, 2026

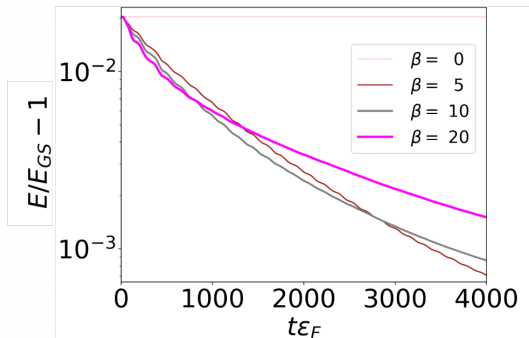
$$\Delta_{\text{cool}} = -\beta \sin[\theta_B - \theta_\chi] \Delta_{\text{MF}},$$

β : controls the strength of the cooling



$$\dot{E} < 0, \quad \dot{N} = 0$$

keeping an unitary time evolution



**Warsaw University
of Technology**

**W-SLDA Toolkit
W-BSk Toolkit**

W-SLDA Toolkit

*Self-consistent solver
of mathematical problems
which have structure
formally equivalent to
Bogoliubov-de Gennes equations.*

static problems: st-wsllda

$$\begin{pmatrix} h_a(\mathbf{r}) - \mu_a & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h_b^*(\mathbf{r}) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix}$$

time-dependent problems: td-wsllda

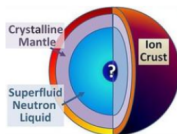
$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_n(\mathbf{r}, t) \\ v_n(\mathbf{r}, t) \end{pmatrix} = \begin{pmatrix} h_a(\mathbf{r}, t) - \mu_a & \Delta(\mathbf{r}, t) \\ \Delta^*(\mathbf{r}, t) & -h_b^*(\mathbf{r}, t) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}, t) \\ v_n(\mathbf{r}, t) \end{pmatrix}$$

Extension to nuclear matter
in neutron stars

Unified solvers for static and
time-dependent problems

Dimensionalities of
problems: 3D, 2D and 1D

Extension to nuclear matter in neutron stars



The W-SLDA Toolkit has been expanded to encompass nuclear systems, now available as the W-BSk Toolkit.

D. Pęcak, A. Zdanowicz, N. Chamel, P. Magierski,
G. Włazłowski, Phys. Rev. X 14, 041054 (2024)

Integration with VisIt:
visualization, animation and
analysis tool

Speed-up calculations by
exploiting High Performance
Computing

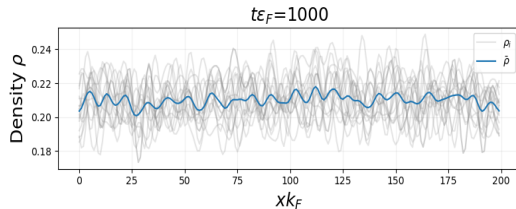
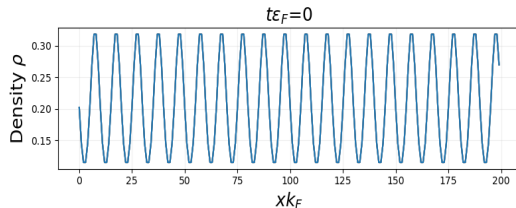
Functionals for studies of
BCS and unitary regimes

ALL FUNCTIONALITIES →

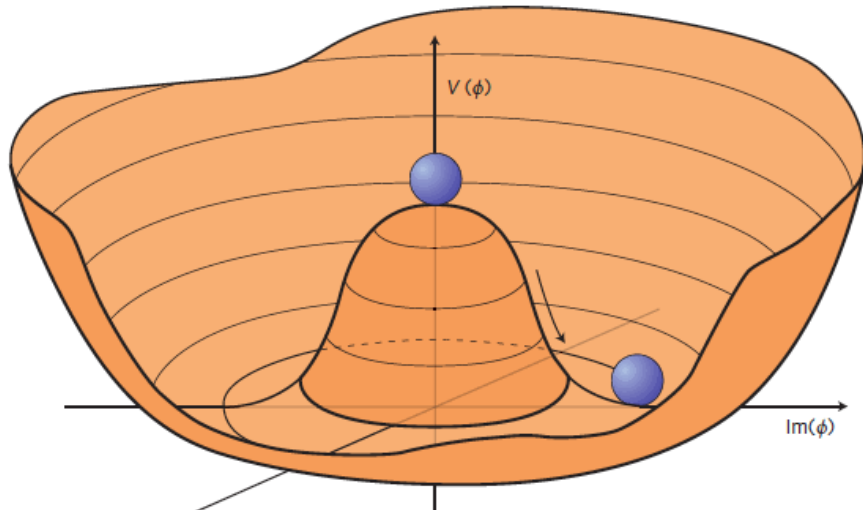
- 1D lattice with $N_x = 200$ with total particle number $N_a = N_b = 21$
- $\frac{\Delta(T=0)}{\epsilon_F} = 0.35$
- The system is excited by a potential of the form:

$$V_{\text{ext}}(x) = A \sin(qx)$$

- 7 initial state with increasing initial energies are evolved
- For all initial states 20 trajectories are made



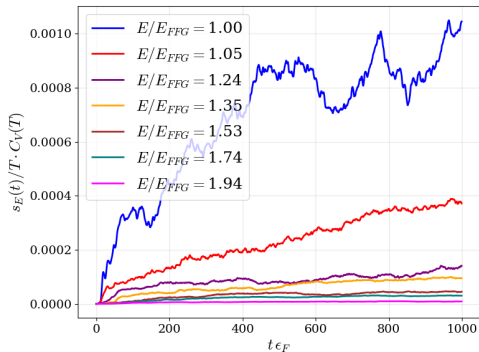
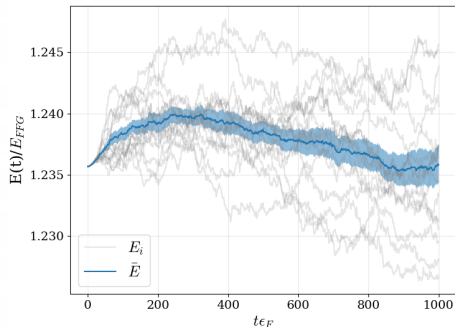
U(1) Symmetry Breaking

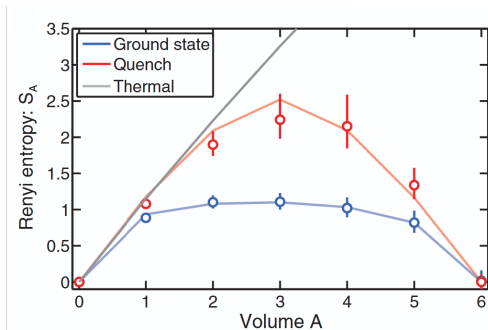
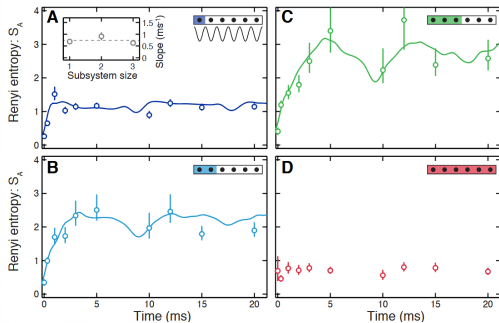


$$E = \int \mathcal{E} dr = \int (\alpha_{\uparrow} \frac{\tau_{\uparrow}}{2} + \alpha_{\downarrow} \frac{\tau_{\downarrow}}{2} + g\nu^+ \nu) dr$$

where $\mathcal{E}$ is the total energy density

$$s_E^2(t) = \frac{1}{N_{\text{tr}} - 1} \sum_{i=1}^{N_{\text{tr}}} [E_i(t) - \bar{E}(t)]^2.$$





- Bose-Einstein condensate of ^{87}Rb atoms loaded into a two-dimensional optical lattice
- They show that entropy thermalize to a fixed value
- The entropy of a smaller subsystem approaches thermal entropy

- For each trajectory a generalized density matrix $\overline{\mathcal{R}}_i$ is computed

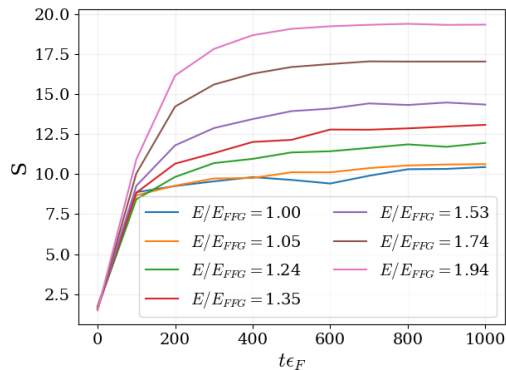
$\Downarrow$

$$\overline{\mathcal{R}} = \frac{1}{M} \sum_i^M \mathcal{R}_i$$

$\Downarrow$

$$\bar{S} = -\frac{1}{2} \sum_i (\lambda_i \ln(\lambda_i) - (1-\lambda_i) \ln(1-\lambda_i))$$

where λ_i are the eigenvalues of $\overline{\mathcal{R}}$



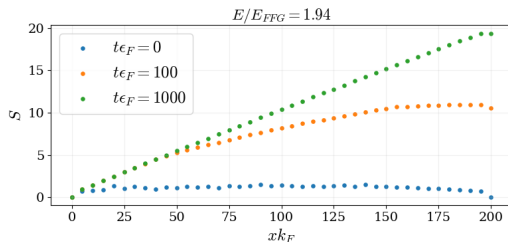
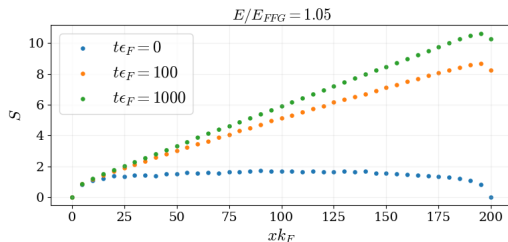
Entropy goes from nonextensive area law, blue dots:

$$S_A \propto \mathcal{A} \log(k_F \xi) + \text{const.}$$

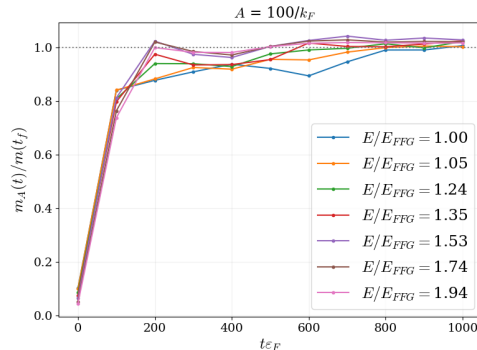
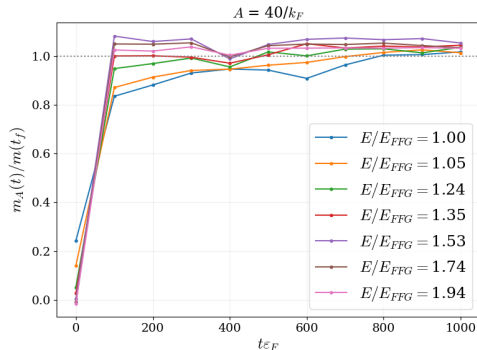
$\Downarrow$

$$S_A(T) \propto s_{th}(T)V_A + \text{const.}$$

to area law, green dots



Subsystem Entropy Plots



m : slope of the entropy linear fit

A : subsystem volume

- We proposed an extension to the BdG equations by adding the stochastic noise to the pairing field
- Together with the quantum cooling we can keep the energy constant with an error small relative to the thermodynamic scale
- We demonstrate that we can test whether we can reach thermalization through two conditions:
 - ▶ By reaching a volume law for the entropy
 - ▶ Smaller subsystems thermalize before the whole system
- Plan to use this extension to a more realistic case and compare with experiments

Thanks for your attention!