

Experimental studies of shape coexistence in $^{94,96}\text{Zr}$

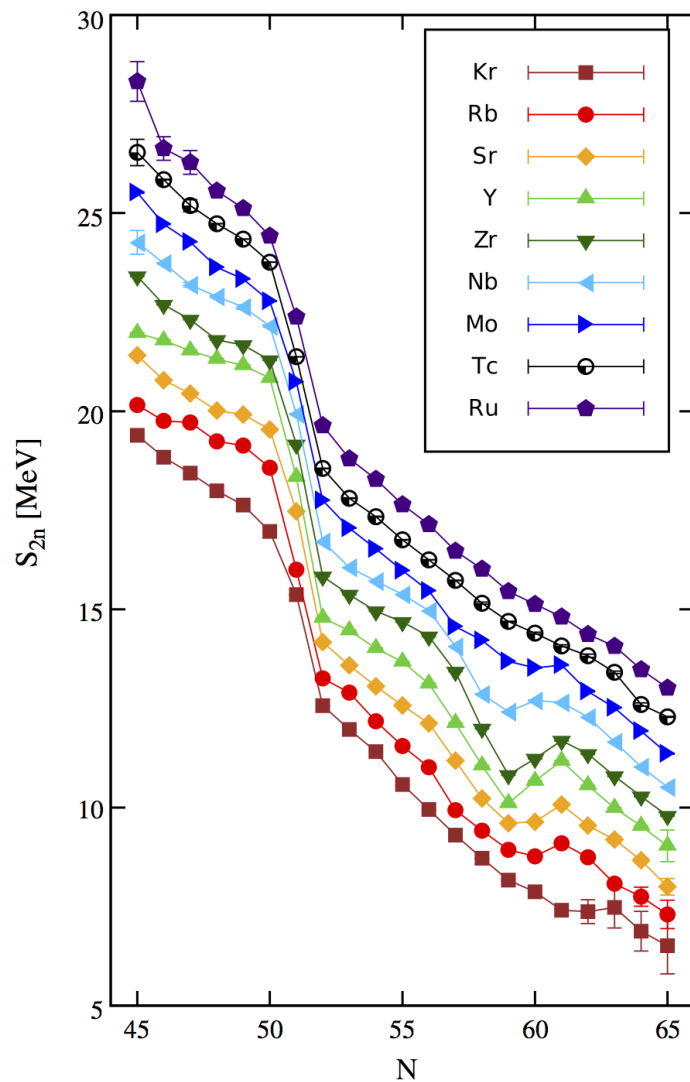
LNL experiment: N. Marchini^{1,2}, M. Rocchini¹, M. Zielińska³, A. Nannini¹, D.T. Doherty⁴, P.E. Garrett⁵, K. Hadyńska-Klęk⁶, A. Goasduff⁷, D. Testov⁸, S.D. Bakes^{4,8}, D. Bazzacco^{8,9}, G. Benzoni¹⁰, T. Berry⁴, D. Brugnara⁷, F. Camera^{10,11}, W.N. Catford⁴, M. Chiari¹, F. Galtarossa⁸, N. Gelli¹, A. Gottardo⁷, A. Gozzelino⁷, A. Illana⁷, J. Keatings¹², D. Mengoni^{8,9}, L. Morrison⁴, D.R. Napoli⁷, M. Ottanelli¹, P. Ottanelli^{1,2}, G. Pasqualato^{8,9}, F. Recchia^{8,9}, S. Riccetto¹³, M. Scheck¹², M. Siciliano^{8,9}, J.J. Valiente Dobon⁷, and I. Zanon^{8,9}

MLL experiment: M. Zielińska³, P.E. Garrett⁵, A. Bergmaier¹⁴, H. Bidaman⁵, V. Bildstein⁵, A. Diaz Varela⁵, D.T. Doherty⁴, T. Fästermann¹⁵, R. Hertzenberger¹⁶, A.T. Laffoley⁵, A.D. MacLean⁵, S. Pannu⁵, M. Rocchini^{1,5}, P. Spagnoletti¹², M. Vandebrouck³, and K. Wrzosek-Lipska⁶

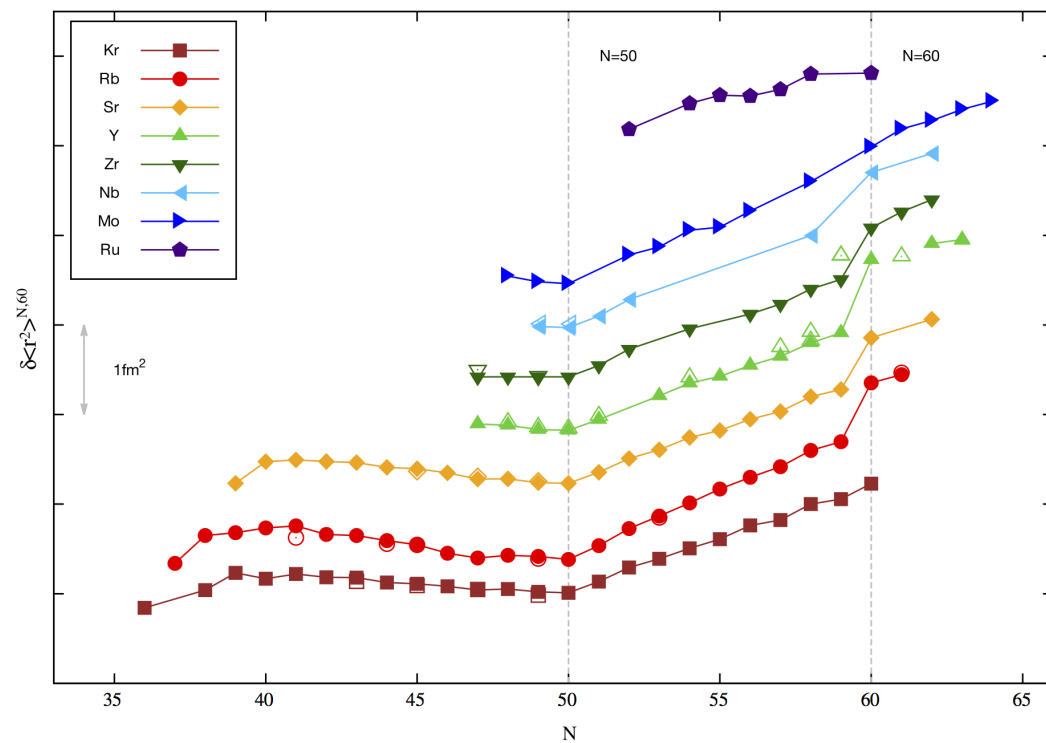
TRIUMF experiment: P.E. Garrett⁵, M. Rocchini^{1,5}, M. Zielińska³, V. Bildstein⁵, A.B. Calder⁵, S.R. Dworatzek⁵, V. Vedia¹⁷, Z.T. Ahmed⁵, C. Andreoiu¹⁸, D. Annen¹⁸, H. Asch¹⁸, A.A. Avaa¹⁷, G. Ball¹⁷, S. Buck⁵, R. Coleman⁵, J.T.H. Dowie¹⁹, A. Garnsworthy¹⁷, E.D. Geerlof¹⁷, B. Greaves⁵, A.L. Grimes¹⁷, G. Hackman¹⁷, S.F. Hicks¹⁹, D. Kalaydjieva⁵, V. Karayonchev²⁰, T. La Marca^{1,2}, S. Lange⁵, B. Lenardo²¹, L. Maqungo⁵, N. Marchini^{1,2}, B. Marlow¹⁷, M.S. Martin¹⁸, K. Mashtakov⁵, E. McCourt⁵, J.R. Murias¹⁷, S. Murillo-Morales¹⁷, A. Nannini¹, S. Pannu⁵, E.E. Peters¹⁹, C. Petrache²², C. Porzio²³, G. Richardson²¹, M. Scheck¹², K. Stoychev⁵, C. Svensson⁵, E. Taddei¹⁸, G. Tocabens²², S. Triambak²⁴, R. Umashankar¹⁷, S. Valbuena⁵, F. Wu¹⁸, and T. Zidar⁵

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Shape transition at N=60

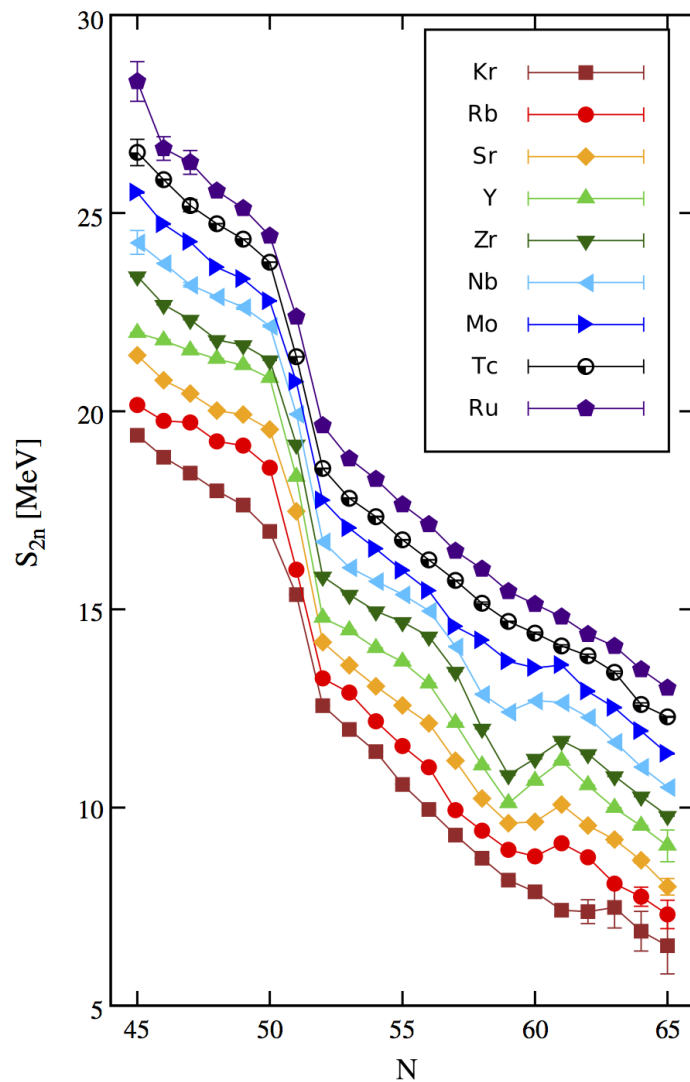


- dramatic change of the ground-state structure observed at $N = 58, 60$ for **Rb**, **Sr**, **Y**, **Zr**
- considerable theoretical and experimental effort in this mass region

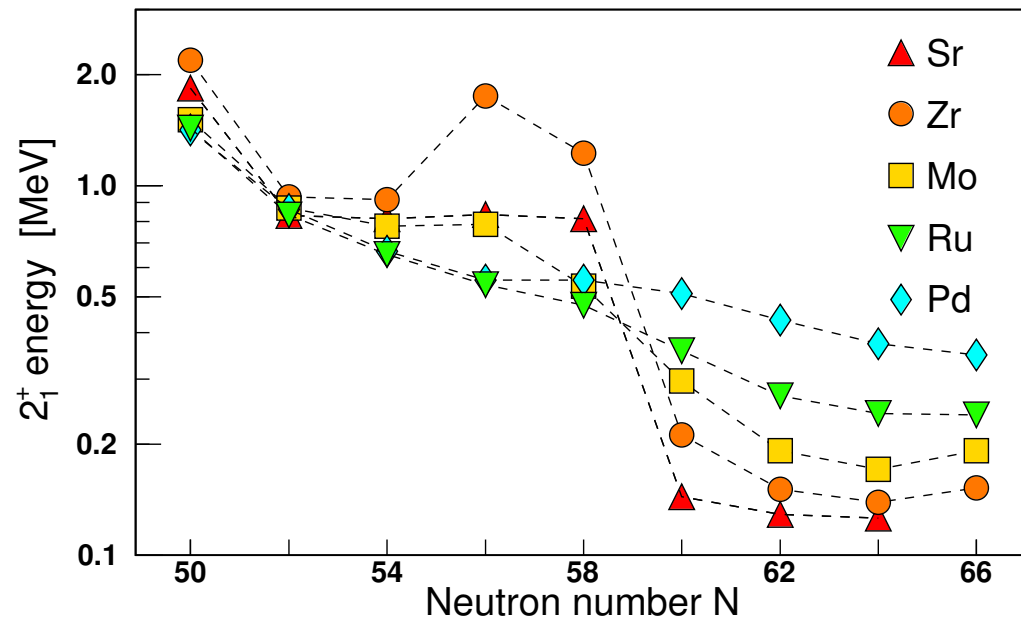


P. Campbell, I. Moore, M. Pearson, Prog. Part. Nucl. Phys. 86 (2016) 127

Shape transition at N=60



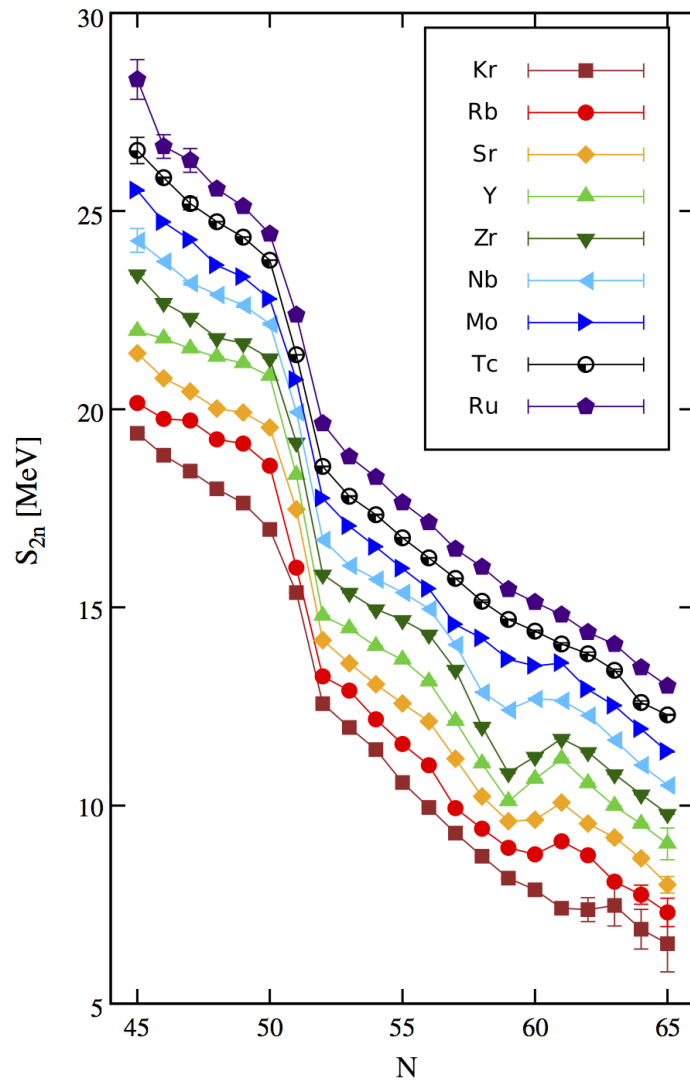
- dramatic change of the ground-state structure observed at $N = 58, 60$ for **Rb**, **Sr**, **Y**, **Zr**
- onset of deformation at $N=60$ confirmed by 2_1^+ energies



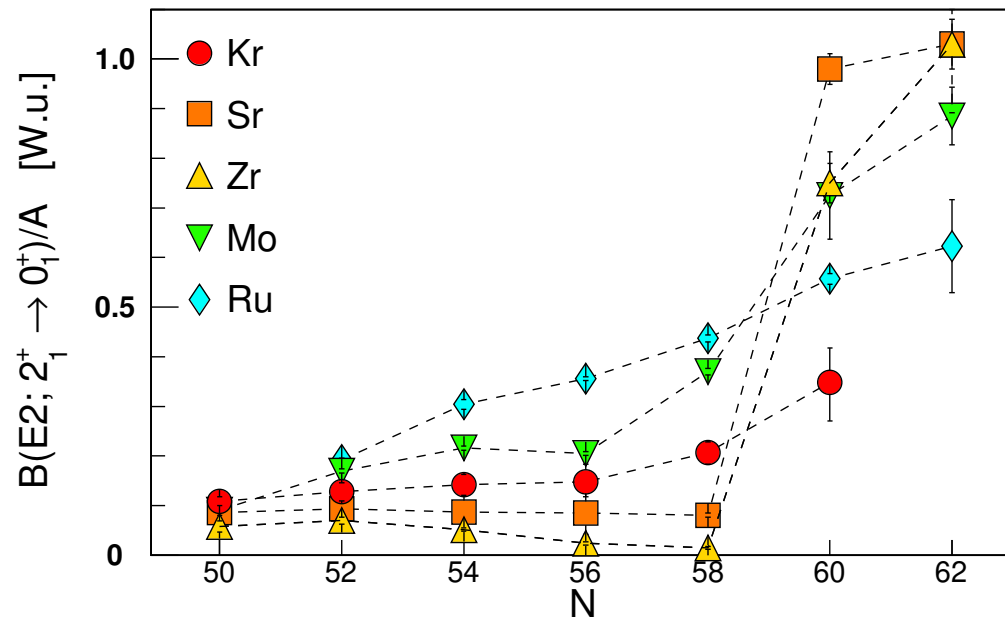
P. Campbell, I. Moore, M. Pearson, Prog. Part. Nucl. Phys. 86 (2016) 127

P. Garrett, MZ, E. Clément, Prog. Part. Nucl. Phys. 124 (2022) 103931

Shape transition at N=60



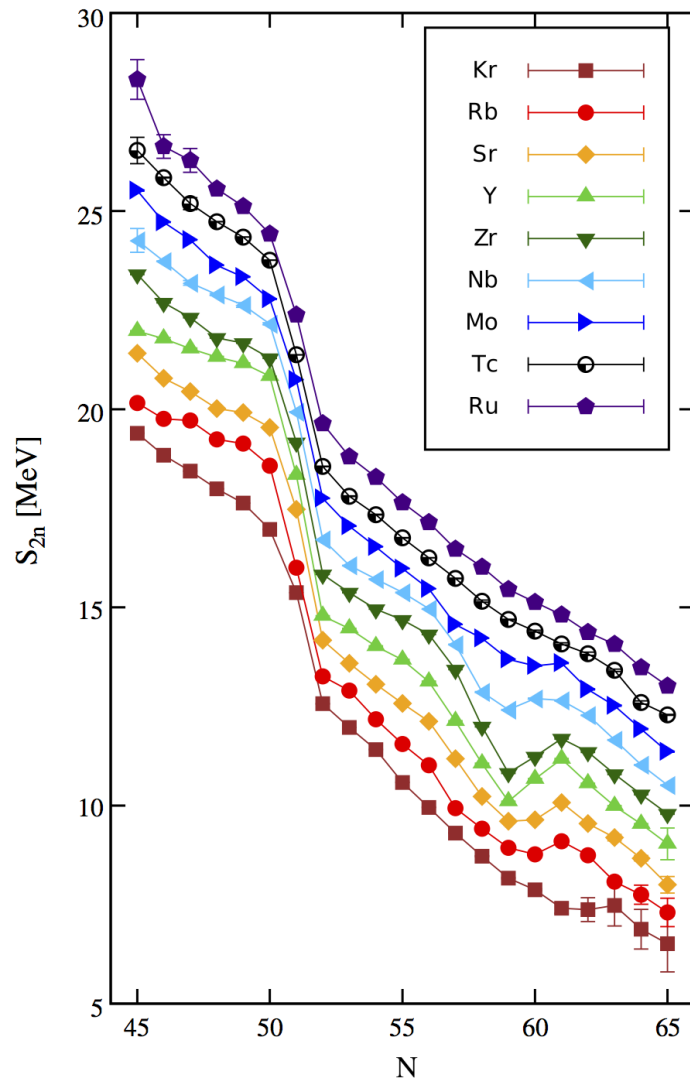
- dramatic change of the ground-state structure observed at $N = 58, 60$ for **Rb**, **Sr**, **Y**, **Zr**
- onset of deformation at $N=60$ confirmed by 2_1^+ energies and transition probabilities



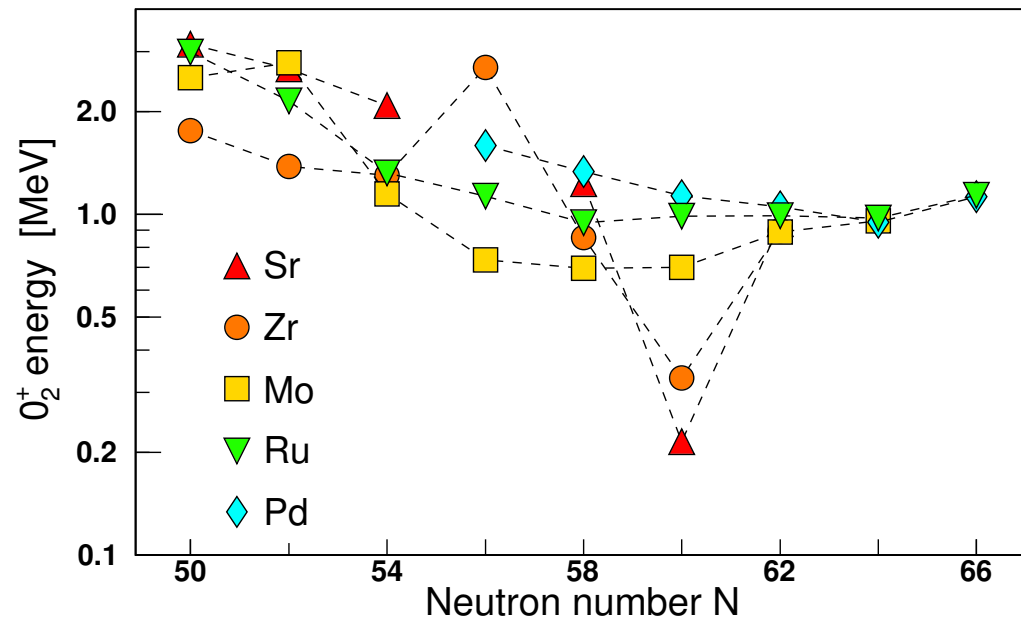
P. Campbell, I. Moore, M. Pearson, Prog. Part. Nucl. Phys. 86 (2016) 127

P. Garrett, MZ, E. Clément, Prog. Part. Nucl. Phys. 124 (2022) 103931

Shape transition at N=60 and shape coexistence around ^{100}Zr



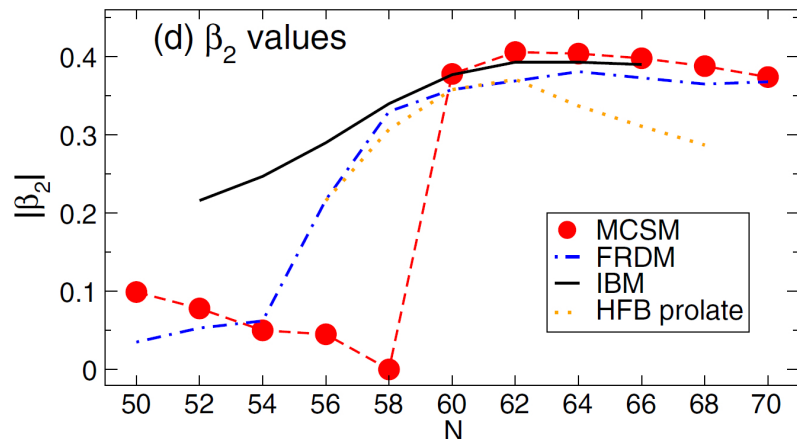
- dramatic change of the ground-state structure observed at $N = 58, 60$ for **Rb**, **Sr**, **Y**, **Zr**
- onset of deformation at $N=60$ confirmed by 2_1^+ energies and transition probabilities
- low-lying 0_2^+ states observed in the Sr, Zr, Mo, Ru isotopes



P. Campbell, I. Moore, M. Pearson, Prog. Part. Nucl. Phys. 86 (2016) 127

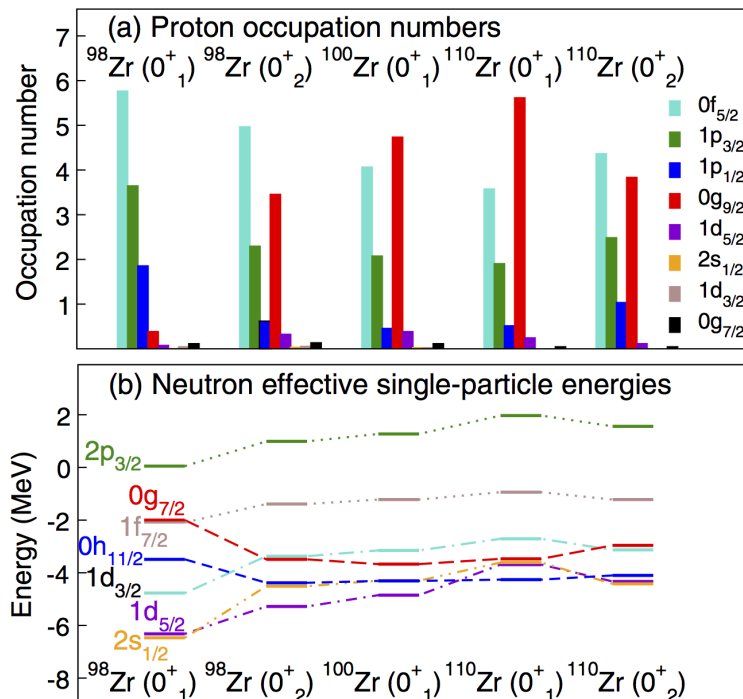
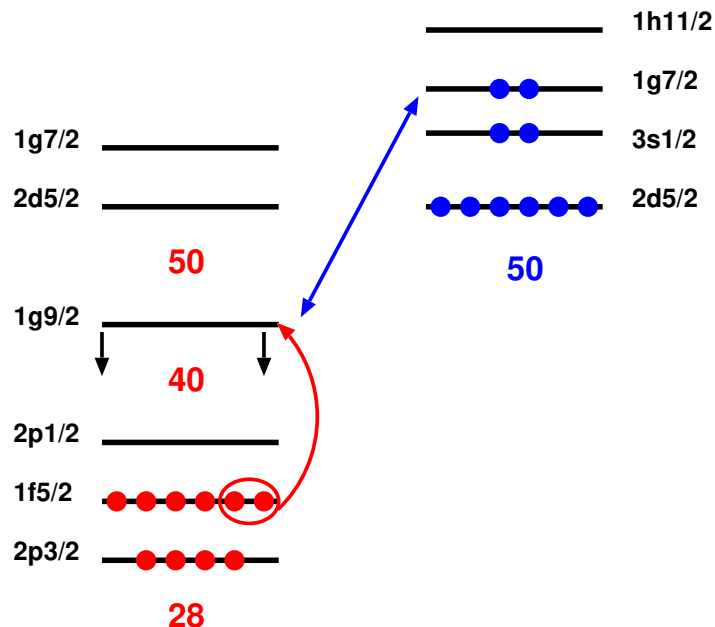
P. Garrett, MZ, E. Clément, Prog. Part. Nucl. Phys. 124 (2022) 103931

Shape coexistence and type-II shell evolution in Zr isotopes



T. Togashi et al, PRL 117, 172502 (2016)

- p-n tensor interaction reduces the $Z=40$ gap when $\nu g_{7/2}$ is being filled
- 0_2^+ states created by 2p-2h (+ 4p-4h...) excitation across $Z=40$
- very different configurations and small mixing of 0_1^+ and 0_2^+

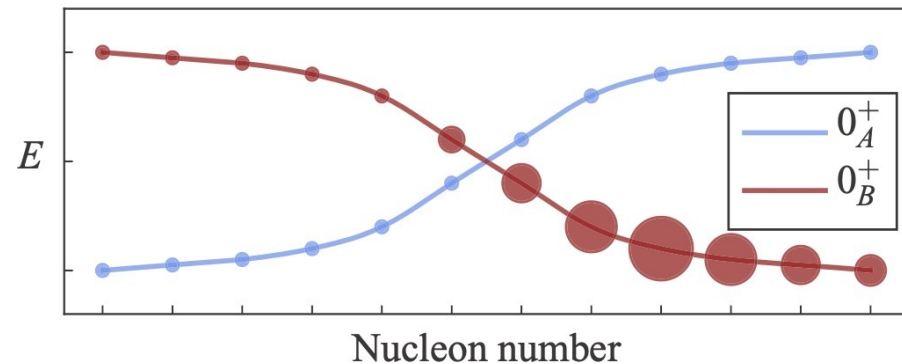


IBM-CM calculations for Zr nuclei

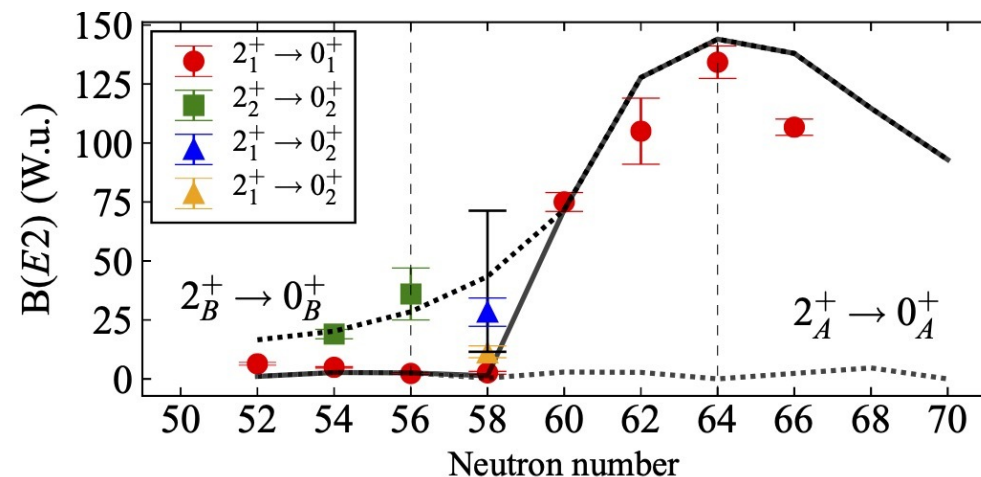
N. Gavrielov et al, PRC 99, 064324 (2019)

N. Gavrielov et al, PRC 105, 014305 (2022)

- “Intertwined quantum phase transition” scenario proposed:
 - two configurations crossing at $N=60$
 - each of them slowly evolving in shape
 - weak mixing between them



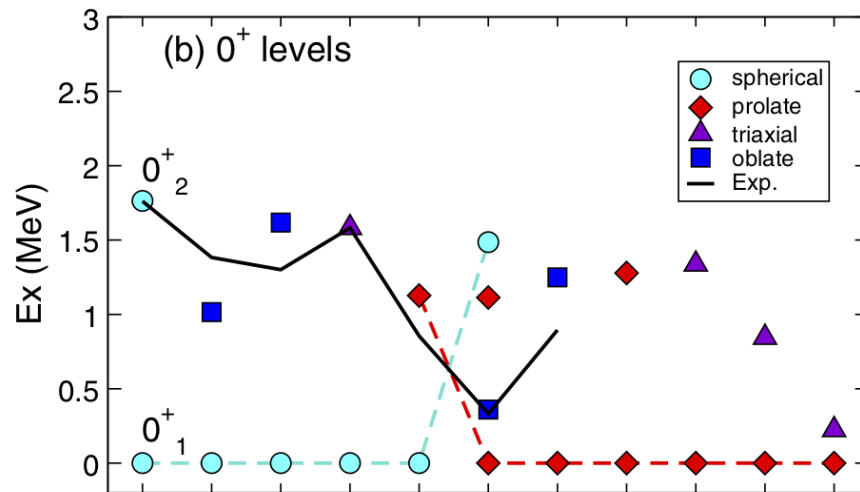
- reproduction of experimental data on Zr nuclei (level energies, $B(E2)$ values, two-neutron separation energies) similarly good as in MCSM



- similar approach successfully applied in: J.E. Ramos-Garcia and K. Heyde, Phys. Rev. C 100, 044315 (2019); Phys. Rev. C 102, 054333 (2020)

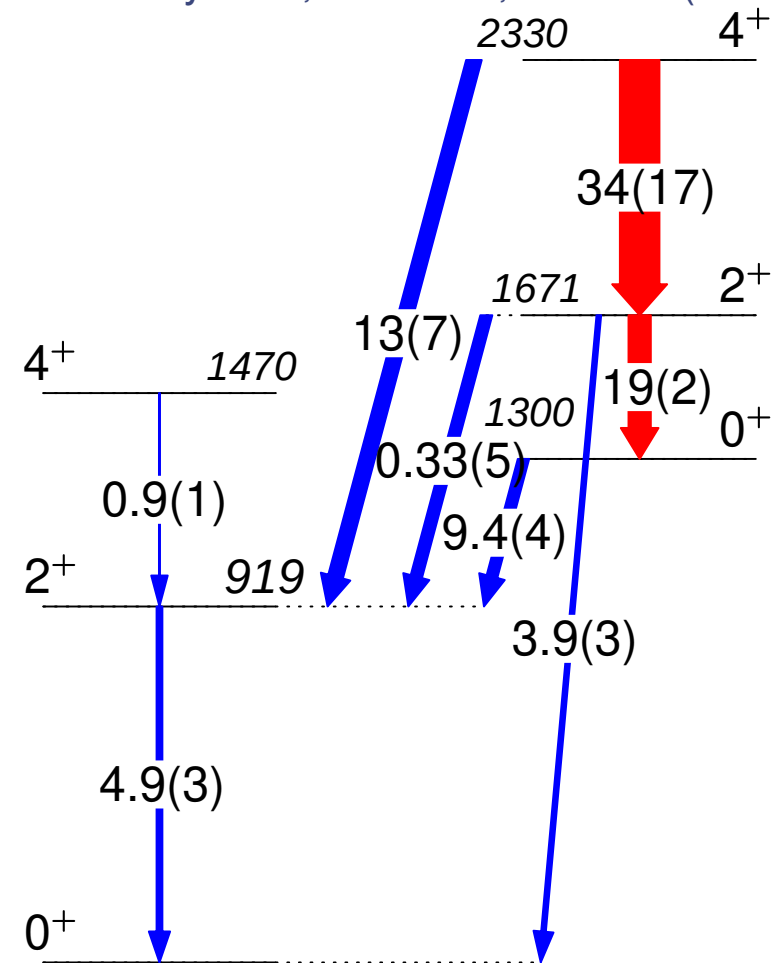
Shape coexistence in ^{94}Zr

A. Chakraborty et al, PRL 110, 022504 (2013)



T. Togashi et al, PRL 117, 172502 (2016)

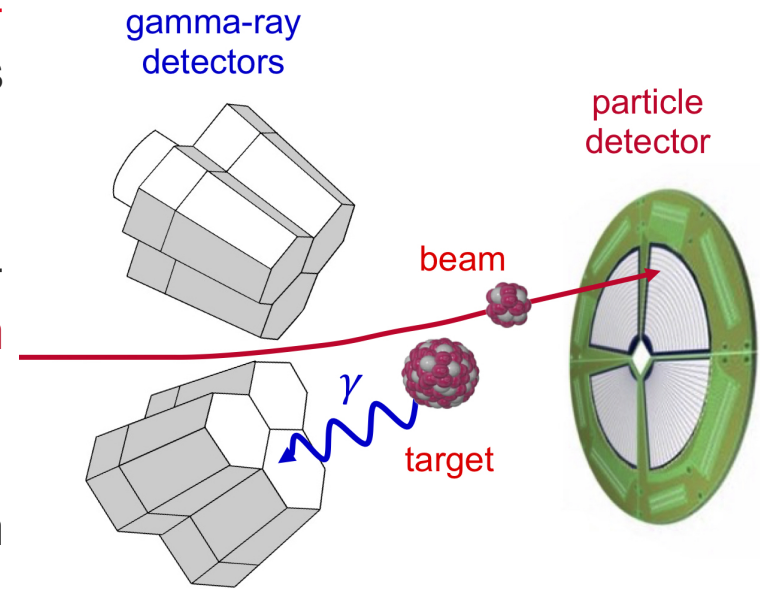
- MCSM calculations suggest a variety of shapes appearing at low excitation energy in Zr nuclei
- ^{94}Zr selected as the first candidate for a detailed experimental investigation
- oblate deformed structure predicted to be built on the 0_2^+ state



- high-statistics β -decay study at TRIUMF: observation of a strong $2_2^+ \rightarrow 0_2^+$ transition (19 W.u.)
 – a deformed band built on 0_2^+

Coulomb excitation

- population of excited states via **purely electromagnetic interaction** between the collision partners in the process of quasi-elastic scattering
- we observe **gamma-ray decay** of Coulomb-excited states in coincidence with **scattered beam ions or target recoils**
- the decay intensities, measured as a function of particle scattering angle, are related to **reduced transition probabilities** and **spectroscopic quadrupole moments** determined via a multi-dimensional fit performed using dedicated analysis codes (e.g. GOSIA)
- they are related to the nuclear shape and collectivity – from extensive sets of E2 matrix elements **quadrupole invariants** can be formed in order to deduce deformation parameters for individual states defined in the intrinsic frame of the nucleus



more about the method: MZ, Lecture Notes in Physics 1005 (2022), chapter 2

Quadrupole sum rules

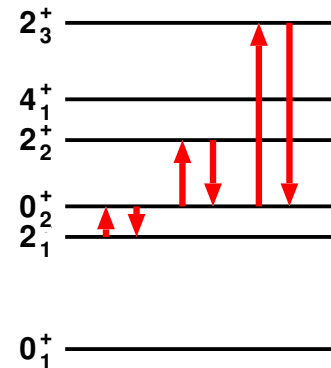
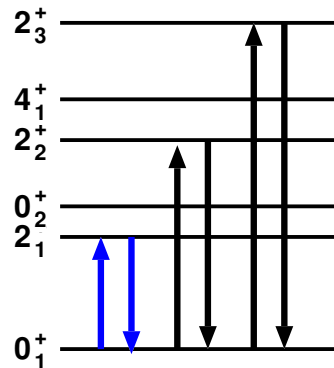
D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683
K. Kumar, PRL 28 (1972) 249

- electromagnetic multipole operators are spherical tensors – products of such operators coupled to angular momentum 0 are rotationally invariant

- in the intrinsic frame of the nucleus, the E2 operator may be expressed using two parameters Q and δ related to charge distribution:

$$\begin{aligned} E(2, 0) &= Q \cos \delta \\ E(2, 2) = E(2, -2) &= \frac{Q}{\sqrt{2}} \sin \delta \\ E(2, 1) = E(2, -1) &= 0 \end{aligned}$$

$$\frac{\langle Q^2 \rangle}{\sqrt{5}} = \langle i | [E2 \times E2]^0 | i \rangle = \frac{1}{\sqrt{(2I_i + 1)}} \sum_t \langle i || E2 || t \rangle \langle t || E2 || i \rangle \begin{Bmatrix} 2 & 2 & 0 \\ I_i & I_i & I_t \end{Bmatrix}$$



$\langle Q^2 \rangle$: measure of the overall deformation;

for the ground state – extension of $B(E2; 0^+ \rightarrow 2^+) = ((3/4\pi)eZR_0^2)^2 \beta_2^2$

Contributions to $\langle Q^2 \rangle$ in ^{100}Mo : K. Wrzosek-Lipska *et al.*, PRC 86 (2012) 064305

Quadrupole sum rules: triaxiality

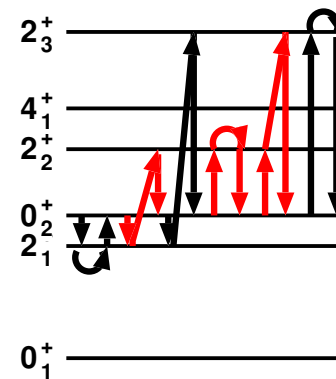
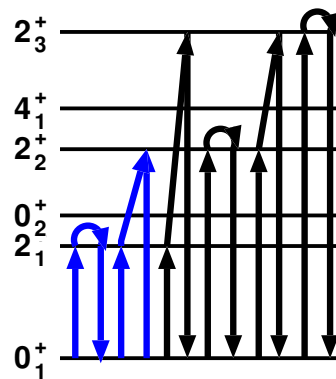
D. Cline, Ann. Rev. Nucl. Part. Sci. 36
(1986) 683

K. Kumar, PRL 28 (1972) 249

$\langle \cos 3\delta \rangle$: measure of triaxiality

$$\sqrt{\frac{2}{35}} \langle Q^3 \cos 3\delta \rangle = \langle i | \{ [E2 \times E2]^2 \times E2 \}^0 | i \rangle$$

$$= \frac{1}{(2I_i + 1)} \sum_{t,u} \langle i || E2 || u \rangle \langle u || E2 || t \rangle \langle t || E2 || i \rangle \begin{Bmatrix} 2 & 2 & 2 \\ I_i & I_t & I_u \end{Bmatrix}$$



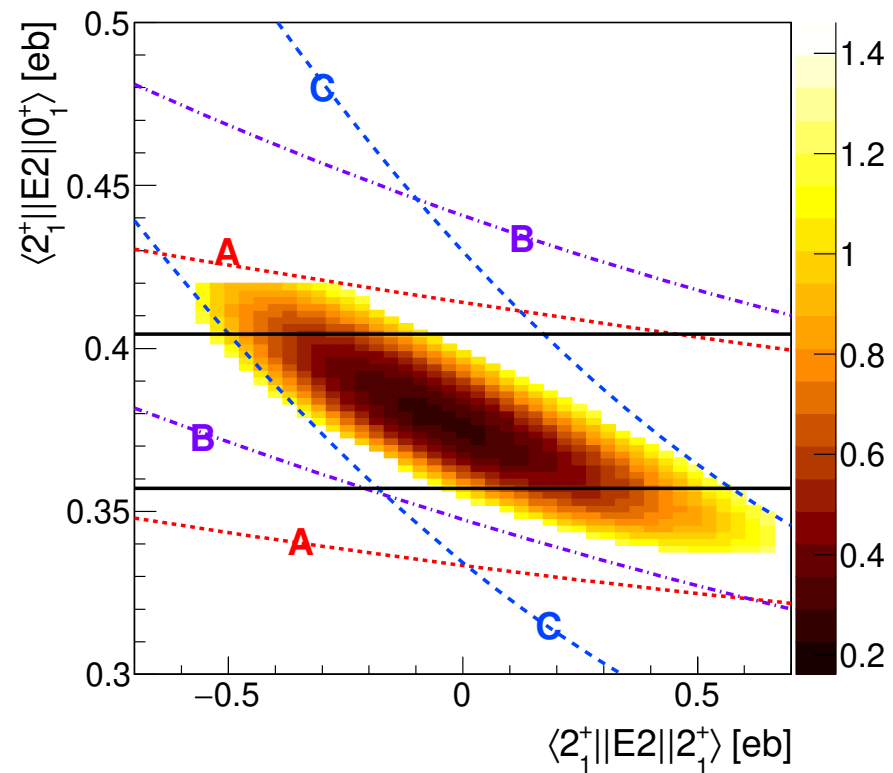
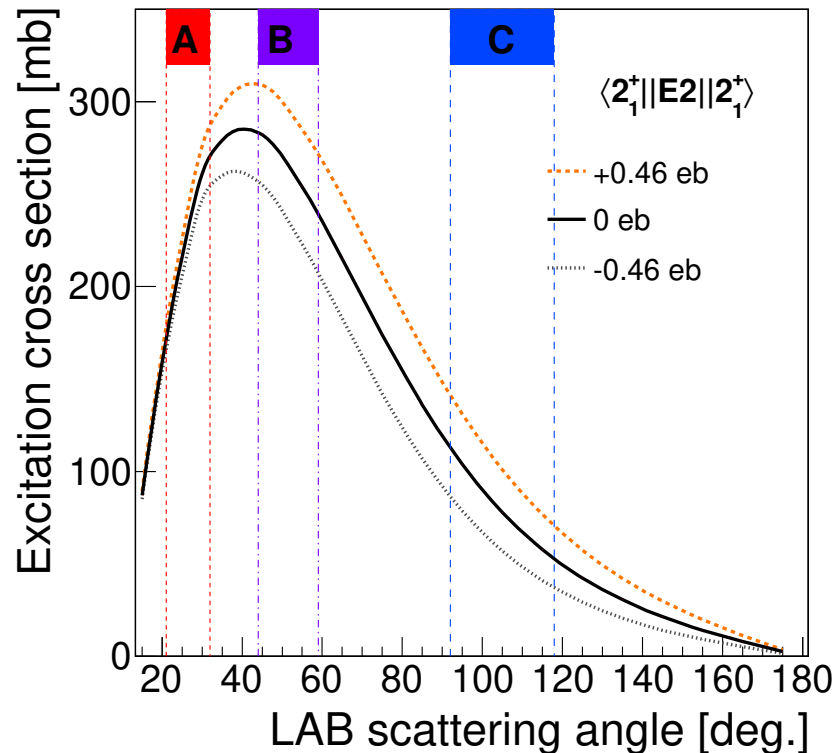
Contributions to $\langle Q^3 \cos 3\delta \rangle$ in ^{100}Mo : K. Wrzosek-Lipska *et al.*, PRC 86 (2012) 064305

- relative signs of E2 matrix elements are needed, as well as quadrupole moments of excited states
- higher-order invariants can a priori be obtained from experimental data, giving information on deformation softness, e.g. $\sigma(Q^2) = \sqrt{\langle Q^4 \rangle - \langle Q^2 \rangle^2}$

Measuring quadrupole moments of excited states

- reorientation effect: influence of the quadrupole moment on the excitation cross section

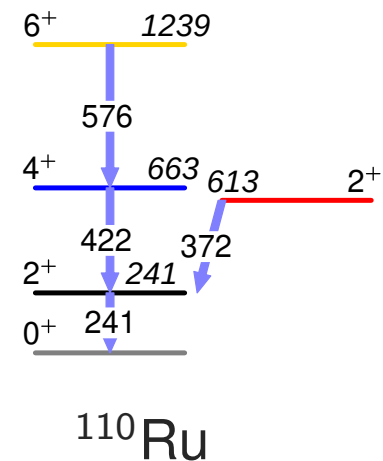
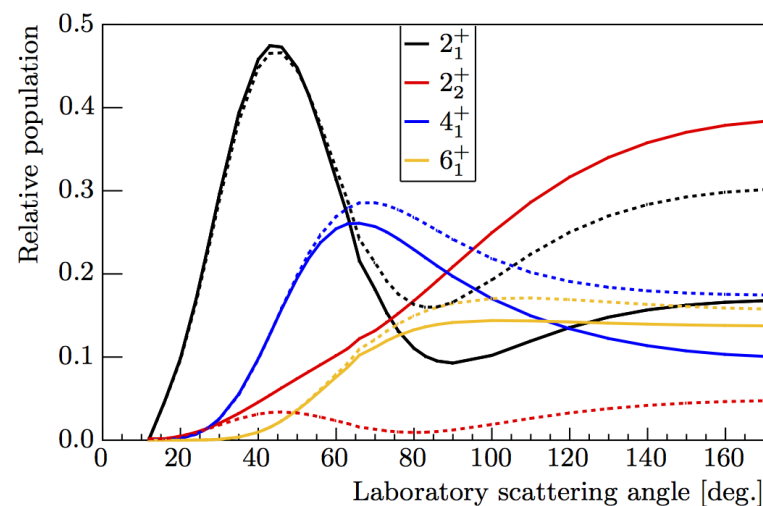
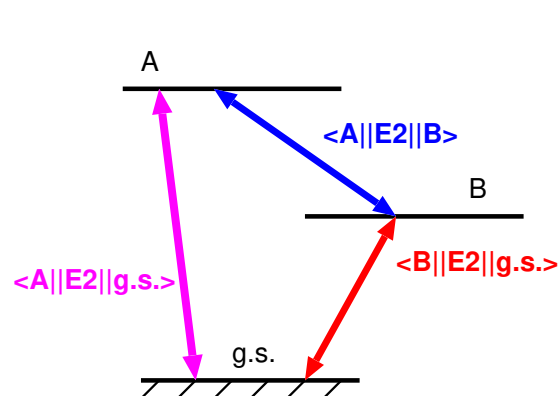
^{76}Zn , HIE-ISOLDE data from: A. Illana, MZ *et al.*, PRC 108, 044305 (2023)



- χ^2 comparison of measured cross sections with calculated ones
- independent lifetime measurements increase precision of extracted quadrupole moments

Relative signs of E2 matrix elements

- Coulomb-excitation cross section are sensitive to relative signs of MEs: result of interference between single-step and multi-step amplitudes
- excitation amplitude of state A: $a_A \sim \langle A || E2 || g.s. \rangle + \langle B || E2 || g.s. \rangle \langle A || E2 || B \rangle$
- excitation probability ($\sim a_A^2$) contains interference terms $\langle A || E2 || g.s. \rangle \langle B || E2 || g.s. \rangle \langle A || E2 || B \rangle$

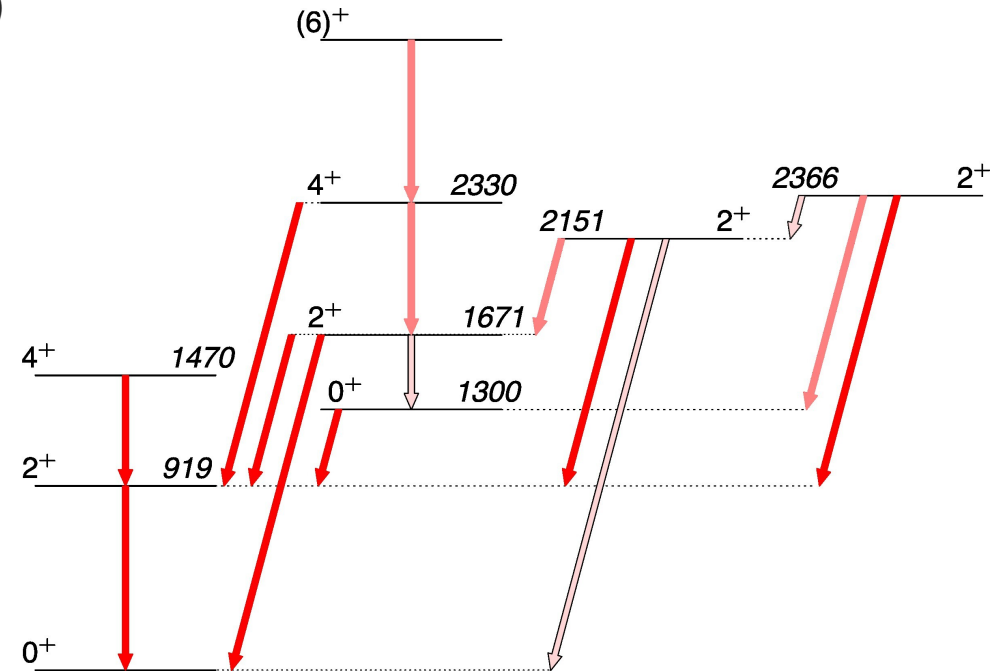
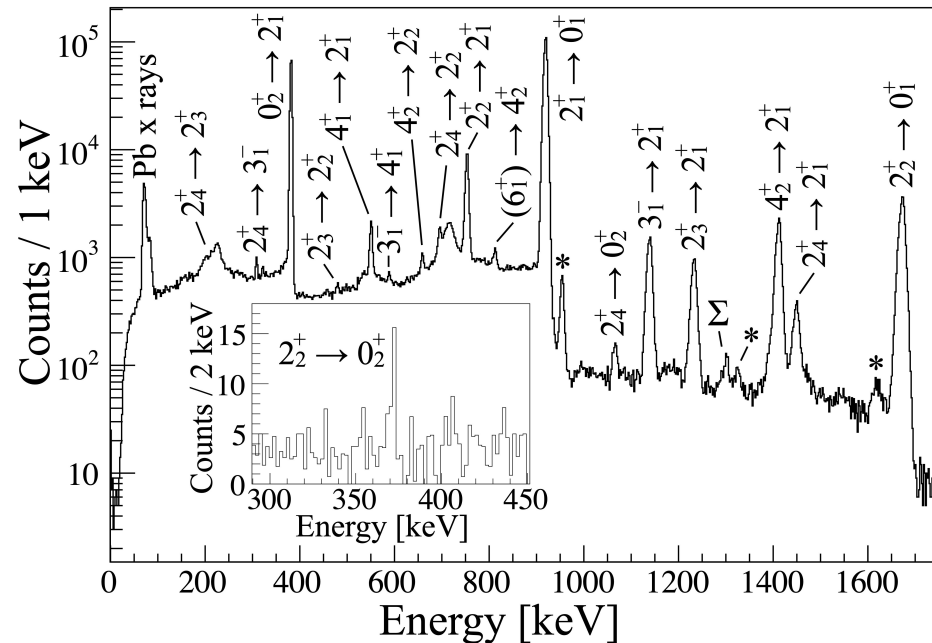


- negative $\langle 2_1^+ || E2 || 2_2^+ \rangle$ (solid lines): much higher population of 2_2^+ at high CM angles
- sign of a product of matrix elements is an observable

Coulomb excitation of ^{94}Zr

N. Marchini et al, PLB 879, 140688 (2026)

- experiment performed at LNL Legnaro: ^{94}Zr beam on ^{208}Pb target
- GALILEO + SPIDER ($121^\circ \leq \theta_{lab} \leq 159^\circ$)



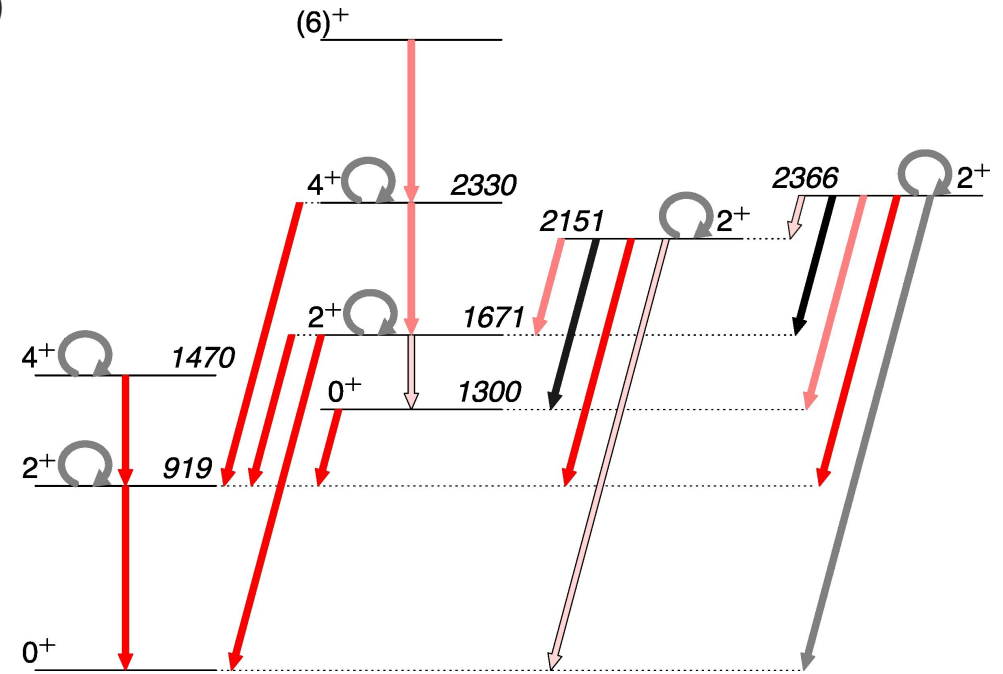
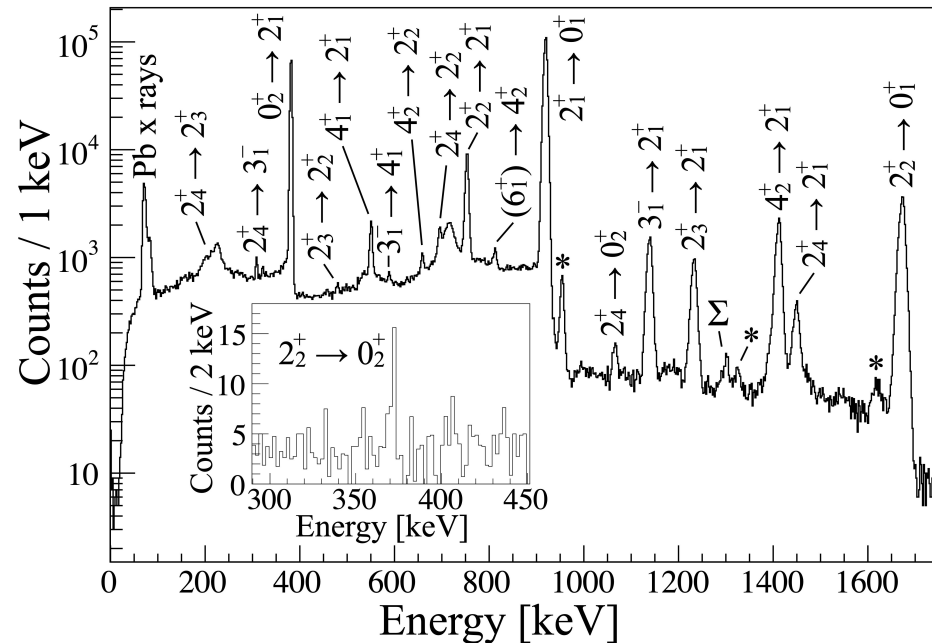
For **several transitions** statistics sufficient to subdivide the data into 8 ranges of scattering angle

For **others** only upper limits could be obtained

Coulomb excitation of ^{94}Zr

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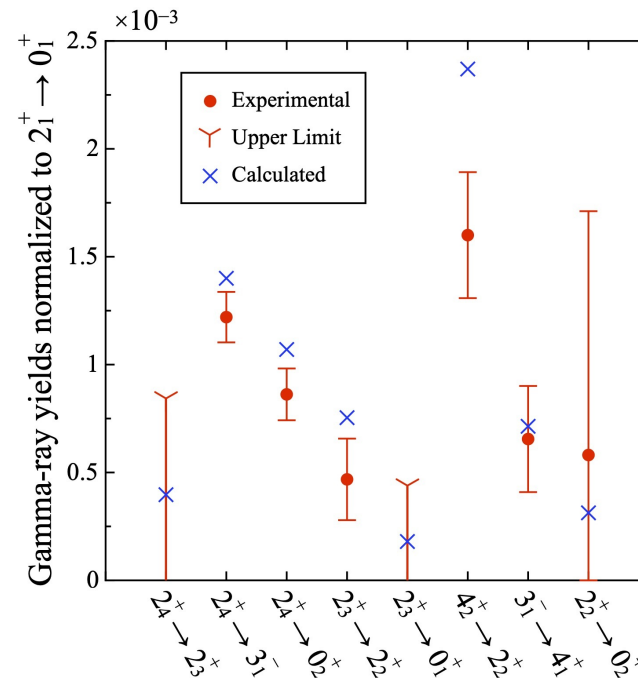
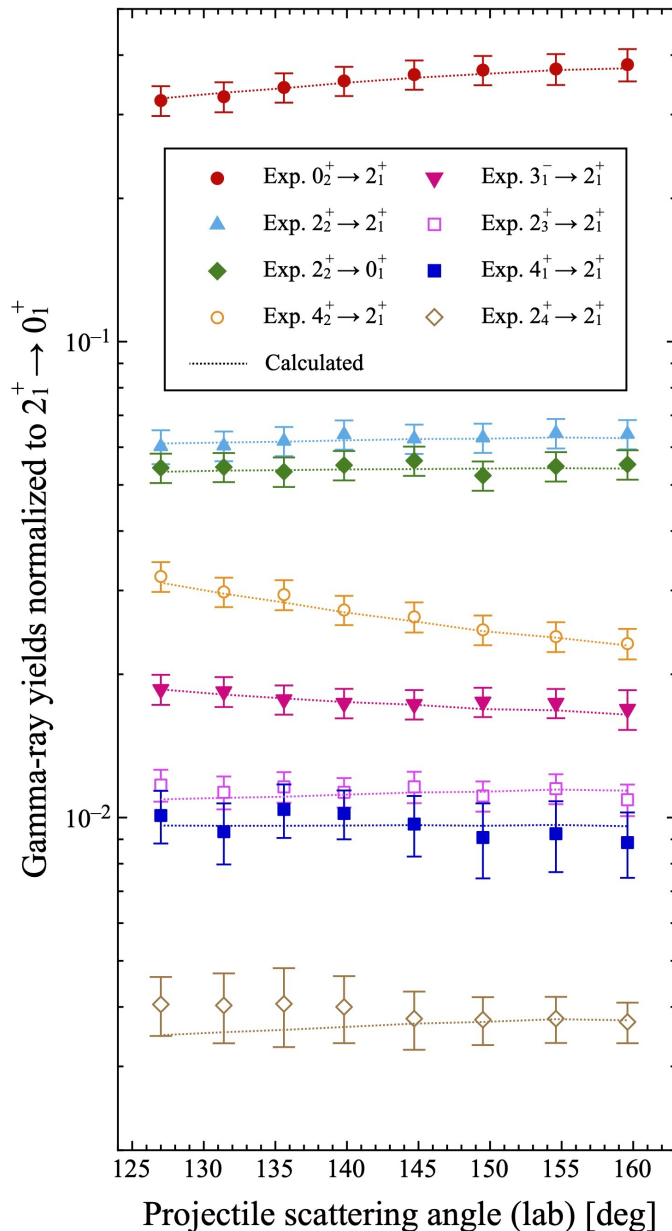
All possible $2_i^+ \rightarrow 2_j^+$ and $2_i^+ \rightarrow 0_j^+$ transitions added to the coupling scheme, as well as static E2 moments

Most branching and mixing ratios known, but not those for $2_4^+ \rightarrow 2_3^+$ and $2_4^+ \rightarrow 0_1^+$

All state lifetimes known, but large uncertainties for 4_2^+ , 2_3^+ , 2_4^+

Fit quality

N. Marchini et al, PLB 879, 140688 (2026)

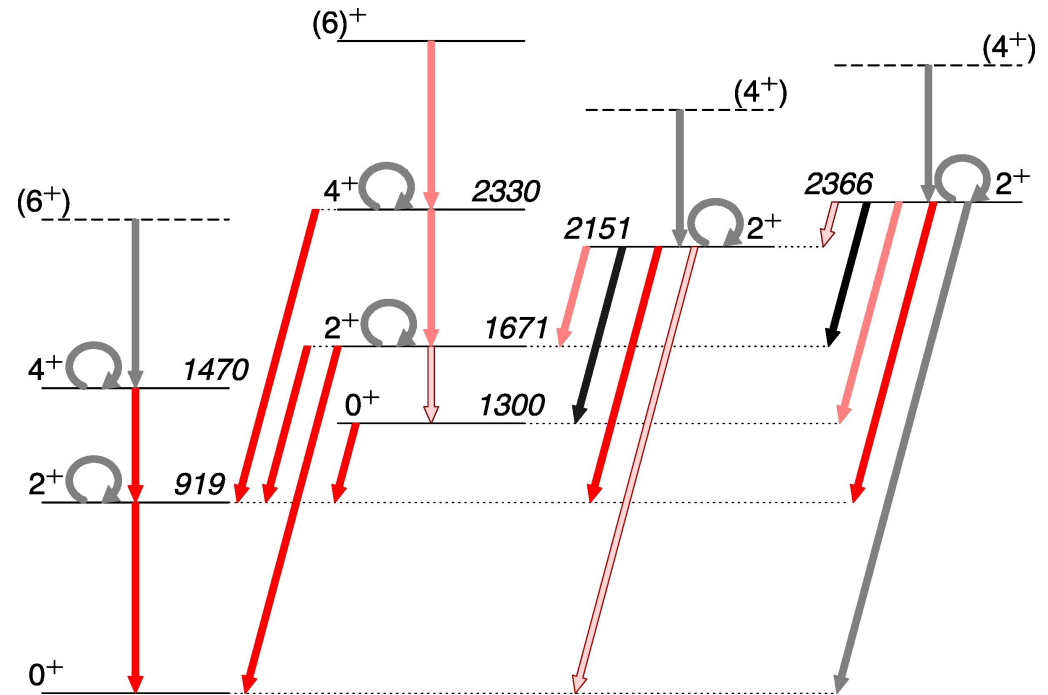
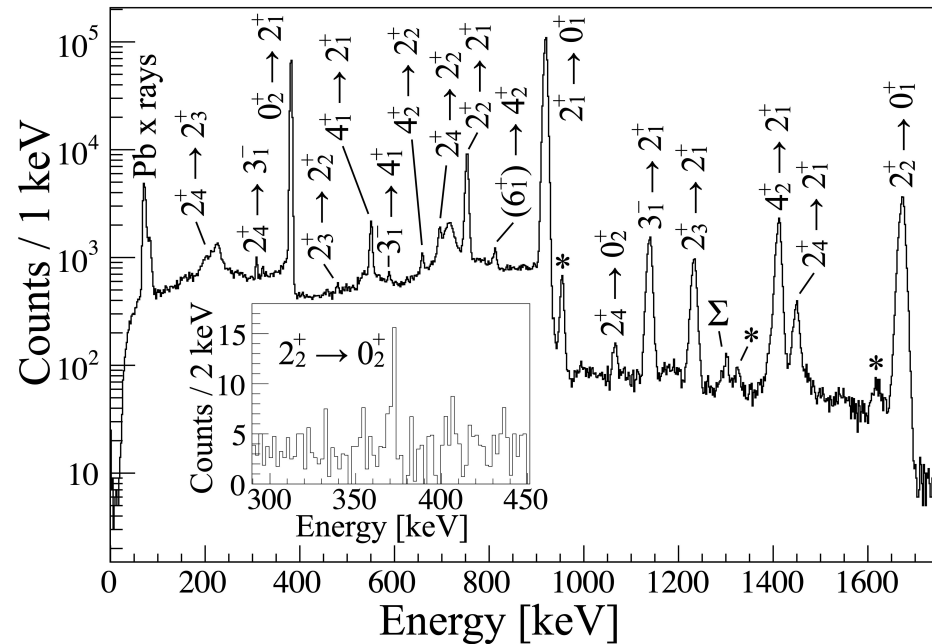


- 32 E2 and M1 matrix elements fitted to 66 γ -ray intensities and 22 lifetimes, branching and mixing ratios
- all literature data reproduced within 1σ except for $\tau(2_4^+)$ (2σ)
- on average, 1.2σ agreement of γ -ray intensities with calculated values
- 19 E2 matrix elements obtained with good precision

Are we happy? Not yet 😞

Alternative solutions

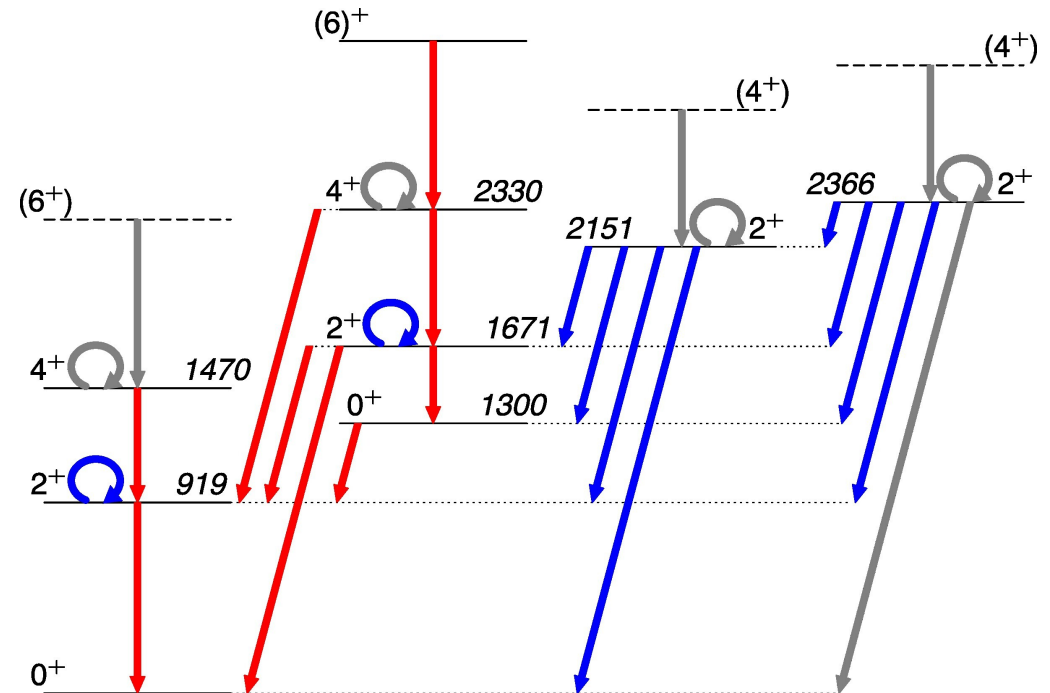
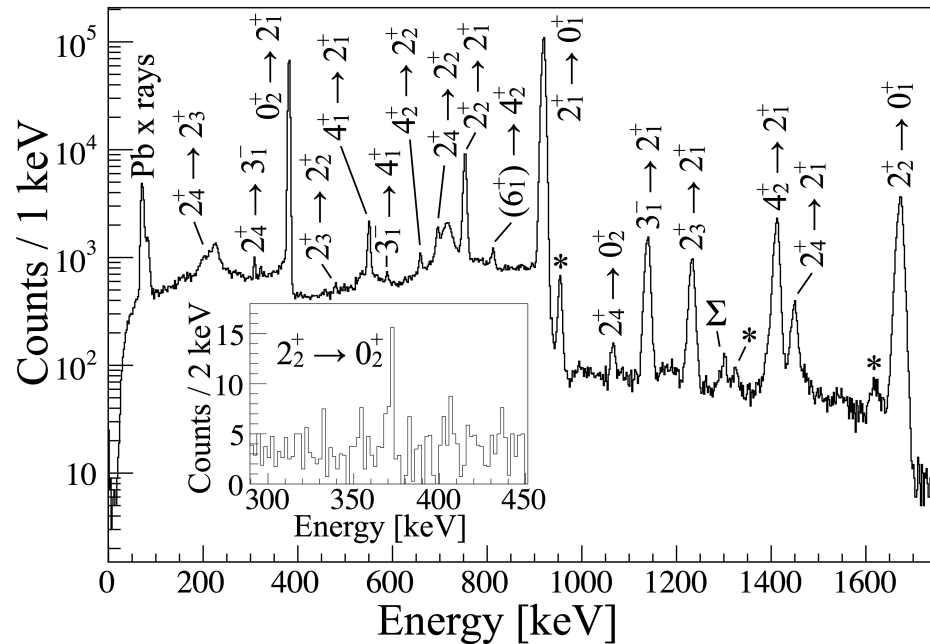
N. Marchini et al, PLB 879, 140688 (2026)



- Worst-case scenario considered for 2_3^+ , 2_4^+ states: we assume that their quadrupole moments may be as large as that of the collective 2_2^+ state, and there may be collective bands built on them
- For the unobserved $2_4^+ \rightarrow 0_1^+$ transition, we consider all possible intensities between 0 and the upper limit deduced from the spectrum

Systematic uncertainties

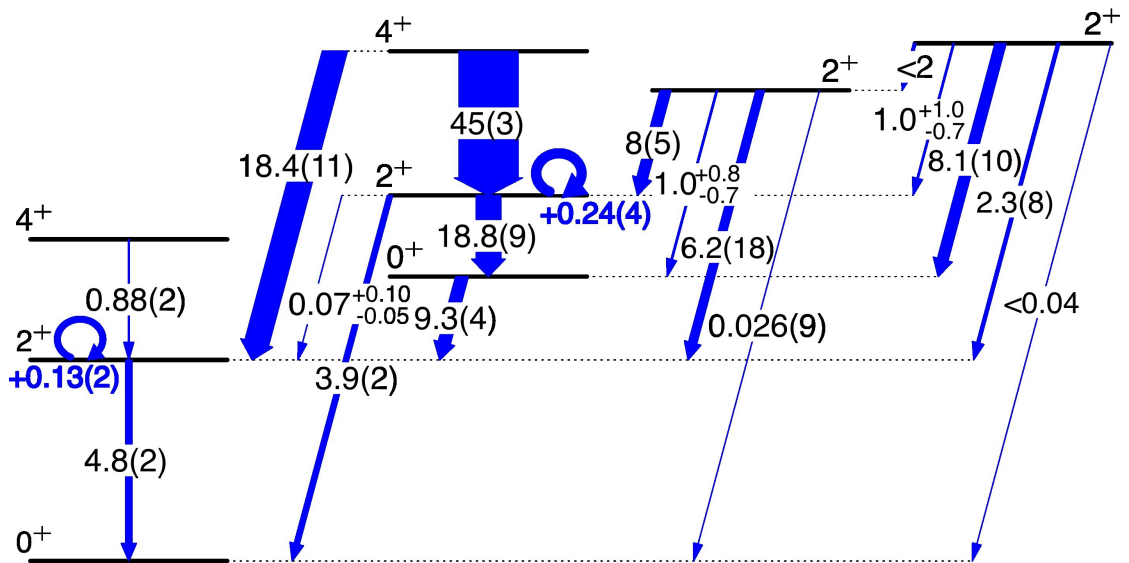
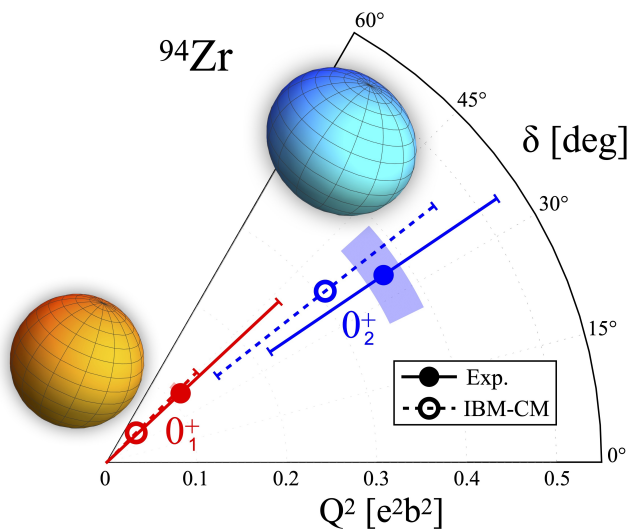
N. Marchini et al, PLB 879, 140688 (2026)



- Worst-case scenario considered for 2_3^+ , 2_4^+ states: we assume that their quadrupole moments may be as large as that of the collective 2_2^+ state, and there may be collective bands built on them
- For the unobserved $2_4^+ \rightarrow 0_1^+$ transition, we consider all possible intensities between 0 and the upper limit deduced from the spectrum
- The systematic uncertainties related to these effects are comparable or larger than statistical ones for **many matrix elements**

Results: quadrupole moments and invariants for ^{94}Zr

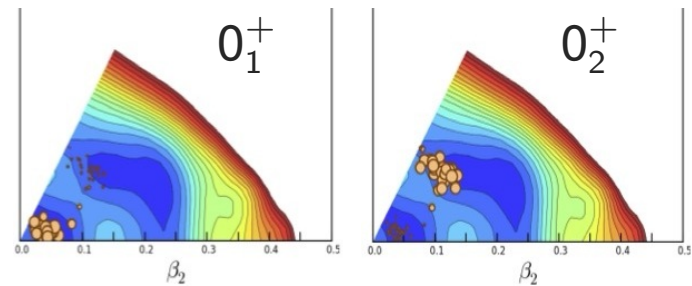
N. Marchini et al, PLB 879, 140688 (2026)



First quadrupole moments measured in even-even Zr nuclei!

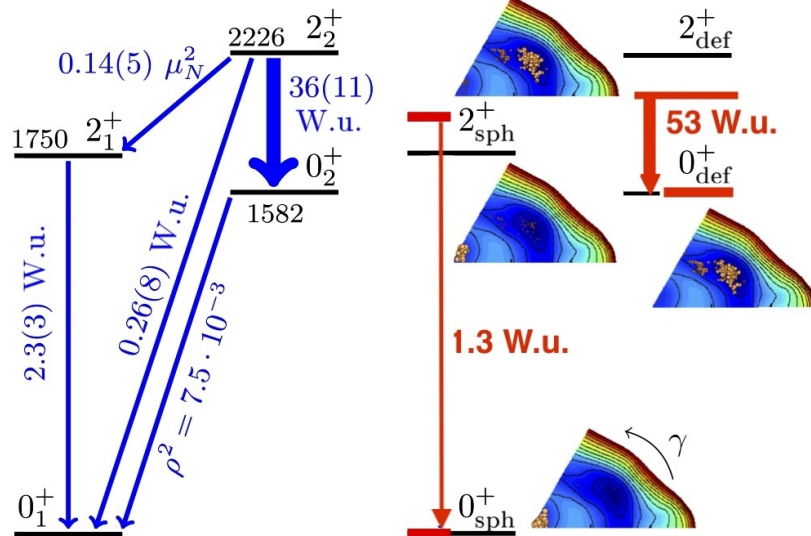
Quadrupole invariants for $0_{1,2}^+$ states:

- spherical shape and softness of the 0_1^+ state
- more rigid, oblate deformed 0_2^+ state ($\langle \beta_2 \rangle \approx 0.2$)

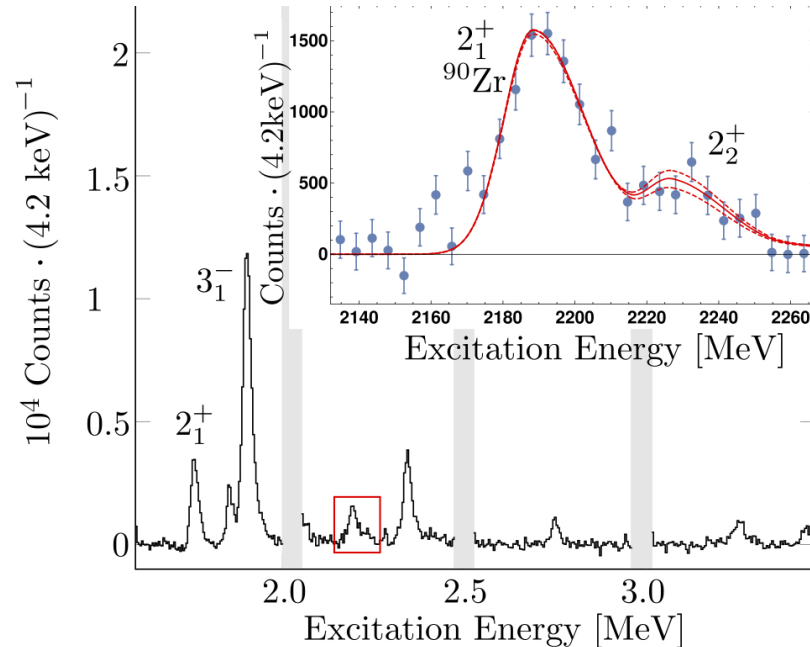


MCSM calculations: T. Otsuka, in: S. Leoni et al, Prog. Nucl. Part. Phys. 104119 (2024)

Shape coexistence in ^{96}Zr – experimental information

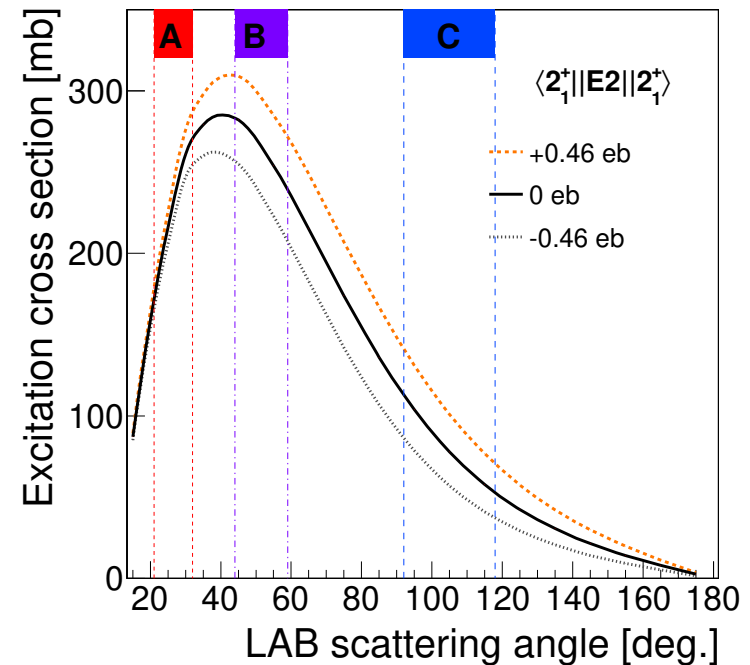
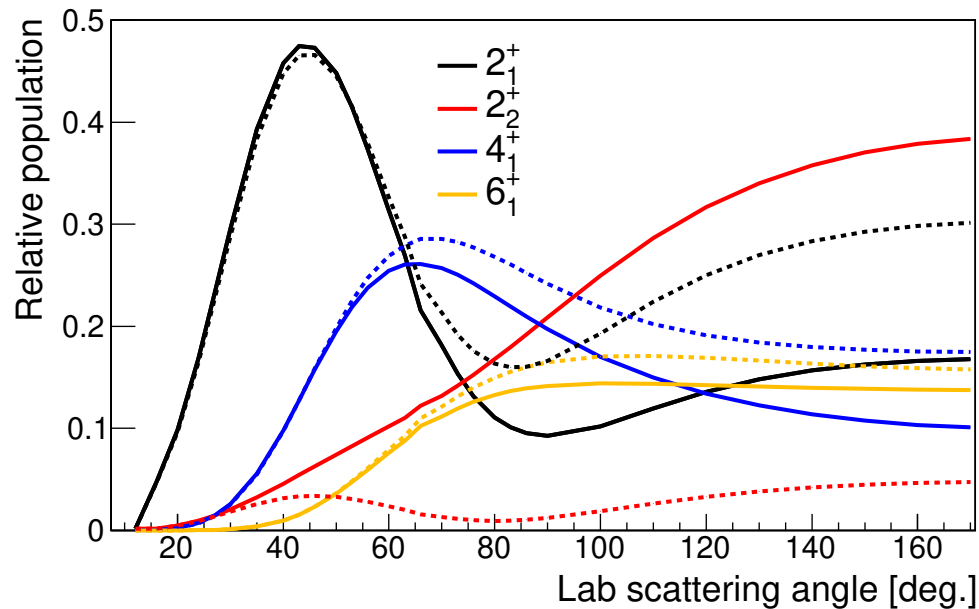


S. Kremer et al, Phys. Rev. Lett. 117 (2017) 172503



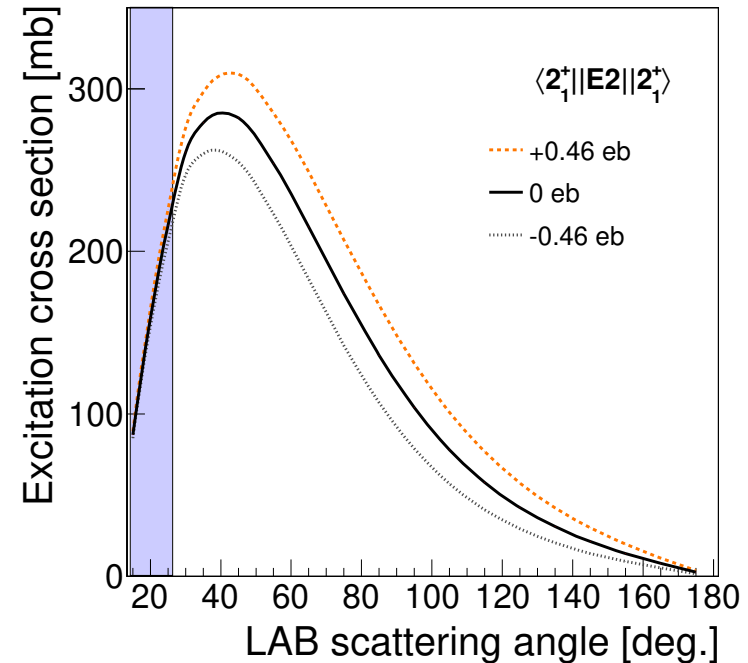
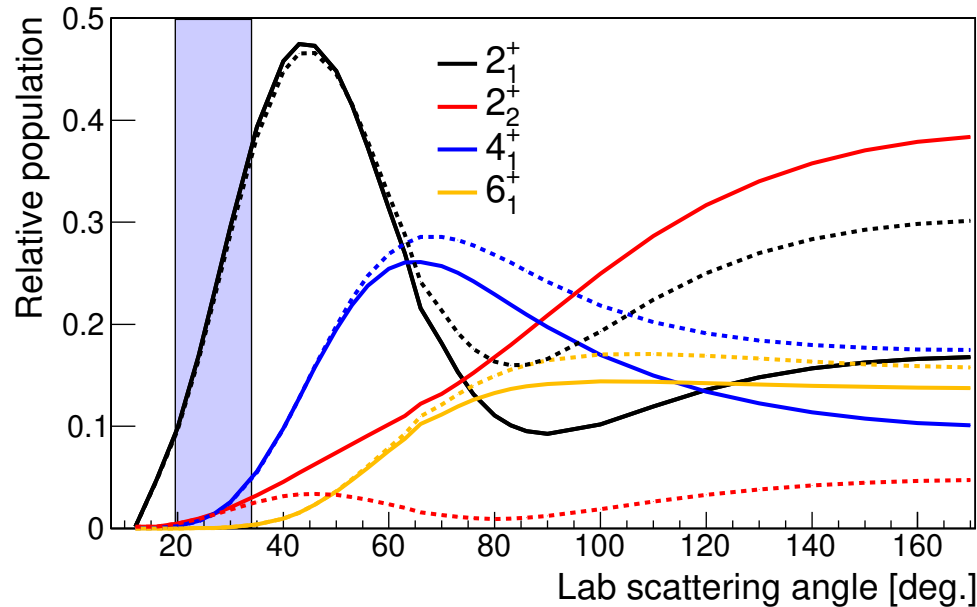
- $B(E2; 2_2^+ \rightarrow 0_1^+)$ measured using electron scattering, combined with known branching and mixing ratios:
→ transition strengths from the 2_2^+ state
- $B(E2; 2_1^+ \rightarrow 0_1^+) = 2.3(3) \text{ Wu}$ vs $B(E2; 2_2^+ \rightarrow 0_2^+) = 36(11) \text{ Wu}$: nearly spherical and a well-deformed structure ($\beta \approx 0.24$)
- very low mixing of coexisting structures: $\cos^2\theta_0=99.8\%$, $\cos^2\theta_2=97.5\%$

How can we improve the precision of $B(E2; 2_2^+ \rightarrow 0_2^+)$ in ^{96}Zr ?



- Influence of relative signs of matrix elements and quadrupole moments increases with scattering angle

How can we improve the precision of $B(E2; 2_2^+ \rightarrow 0_2^+)$ in ^{96}Zr ?

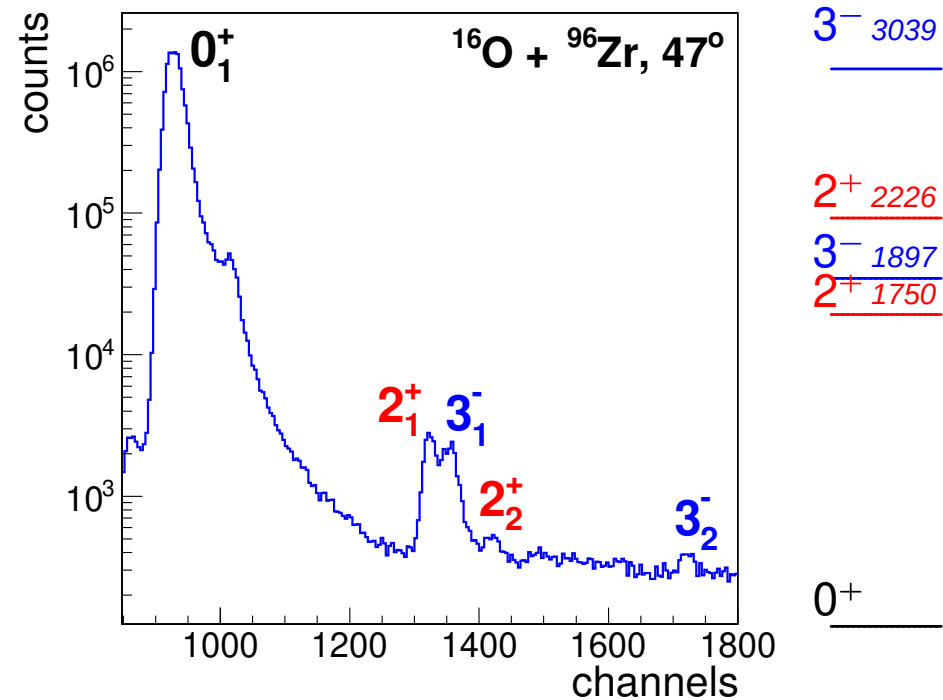
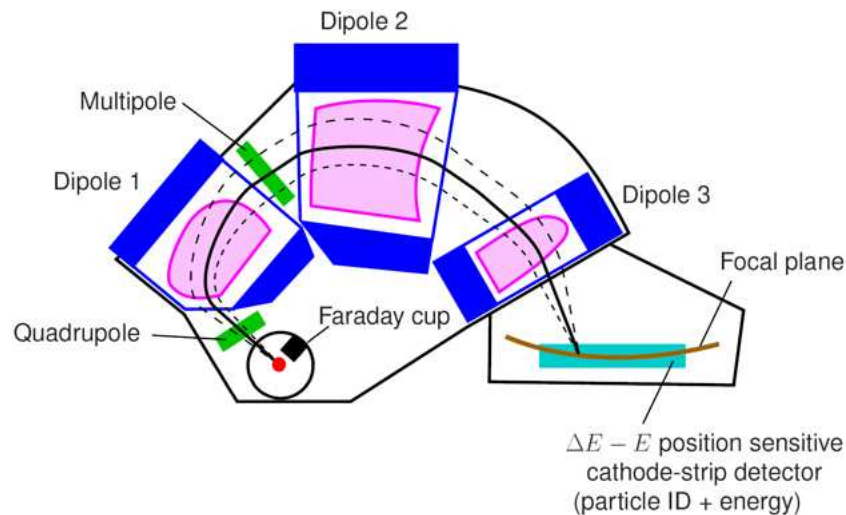


- For low CM angles, cross sections for each state are directly related to $B(E2)$ values for their ground-state transitions (second-order effects strongly limited)
- However, the magnitudes of the cross sections strongly decrease
- If we measure scattered beam particles rather than emitted γ rays, we can compensate for this loss of cross section
 - Higher detection efficiency
 - Possibility of using very high beam currents (100 pA instead of 1 pA)

Coulomb excitation of ^{96}Zr at MLL

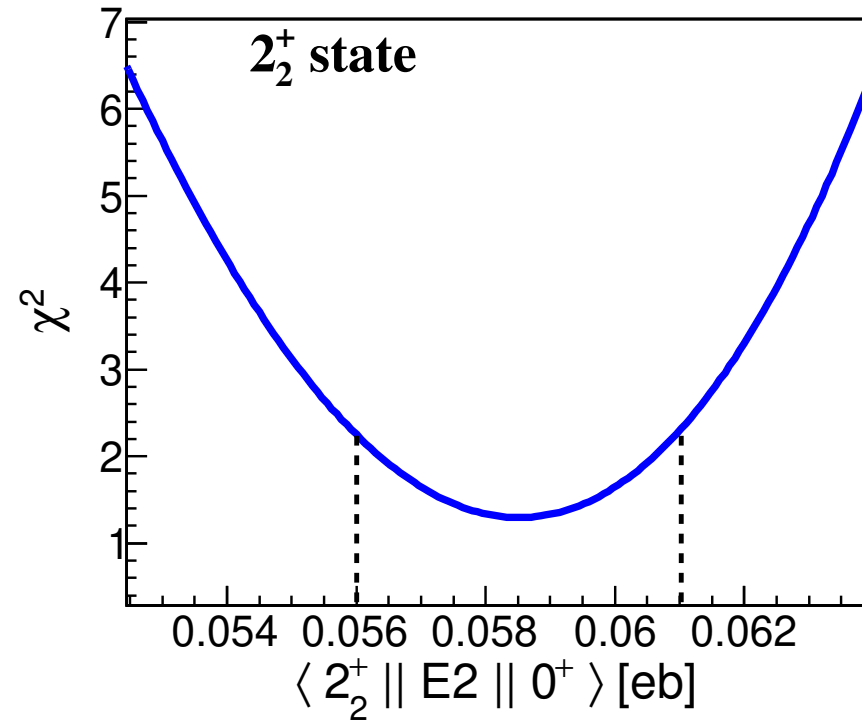
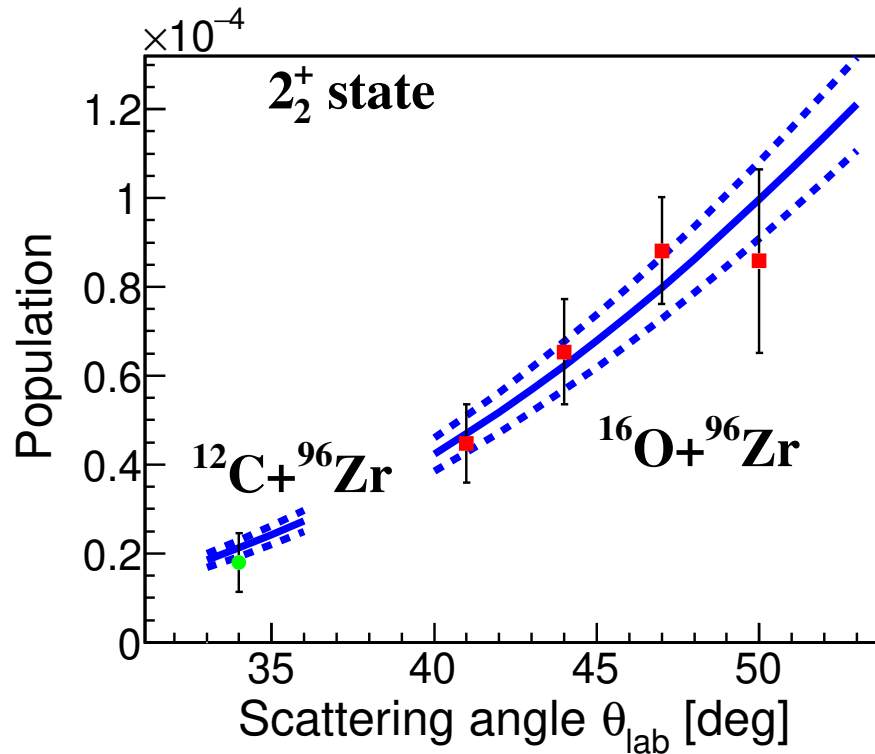
- Coulomb-excitation measurements with magnetic spectrometers common in 1970s, but completely abandoned in favour of γ -ray spectroscopy
- still a very attractive option, especially to populate higher-lying low-spin states
- resolution of ≈ 150 keV – only suitable if level spacing is large enough
- ^{12}C , ^{16}O beams: direct measurement of population of 2^+ states
→ precise $B(E2; 2_1^+ \rightarrow 0_1^+)$ values

Q3D magnetic spectrometer, MLL



Results: $B(E2; 2_{1,2}^+ \rightarrow 0_1^+)$ values in ^{96}Zr

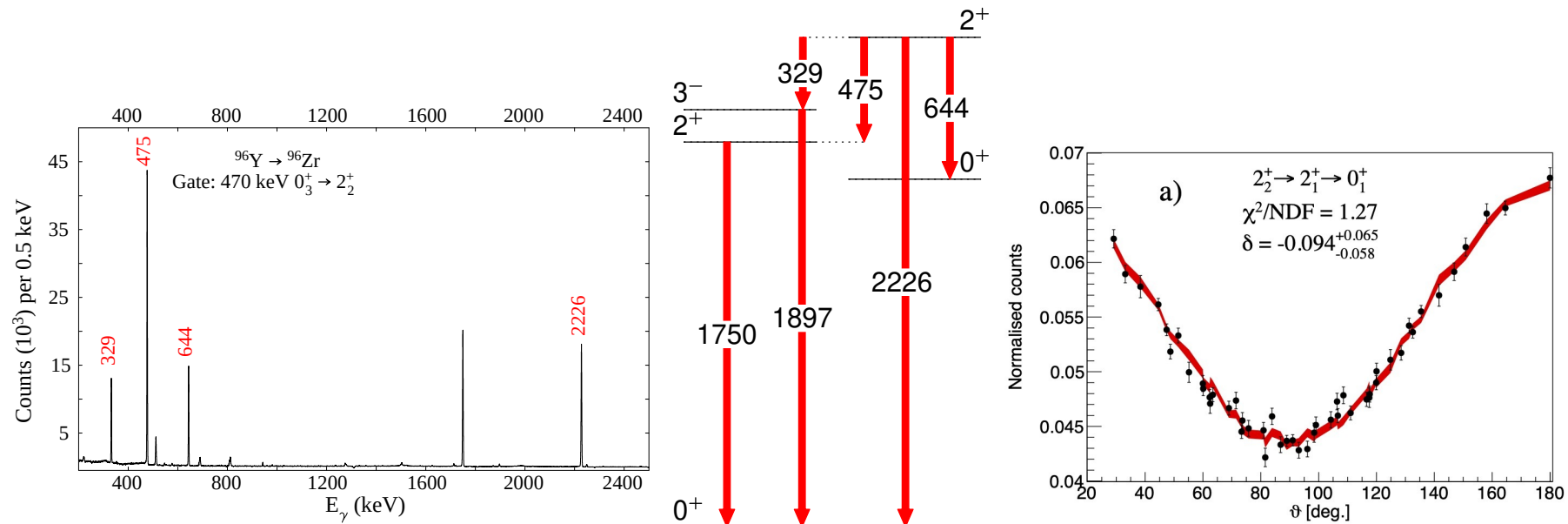
MZ et al, PLB 877, 140481 (2026)



- $B(E2; 2_2^+ \rightarrow 0_1^+) = 0.265(38)$ W.u., versus $0.26(8)$ W.u. from Kremer et al (fully consistent, but $2 \times$ better precision))
- for comparison: $B(E2; 2_1^+ \rightarrow 0_1^+)$ agrees within 1σ with the literature value ($2.73(10)$ W.u. vs $2.3(3)$ W.u.)

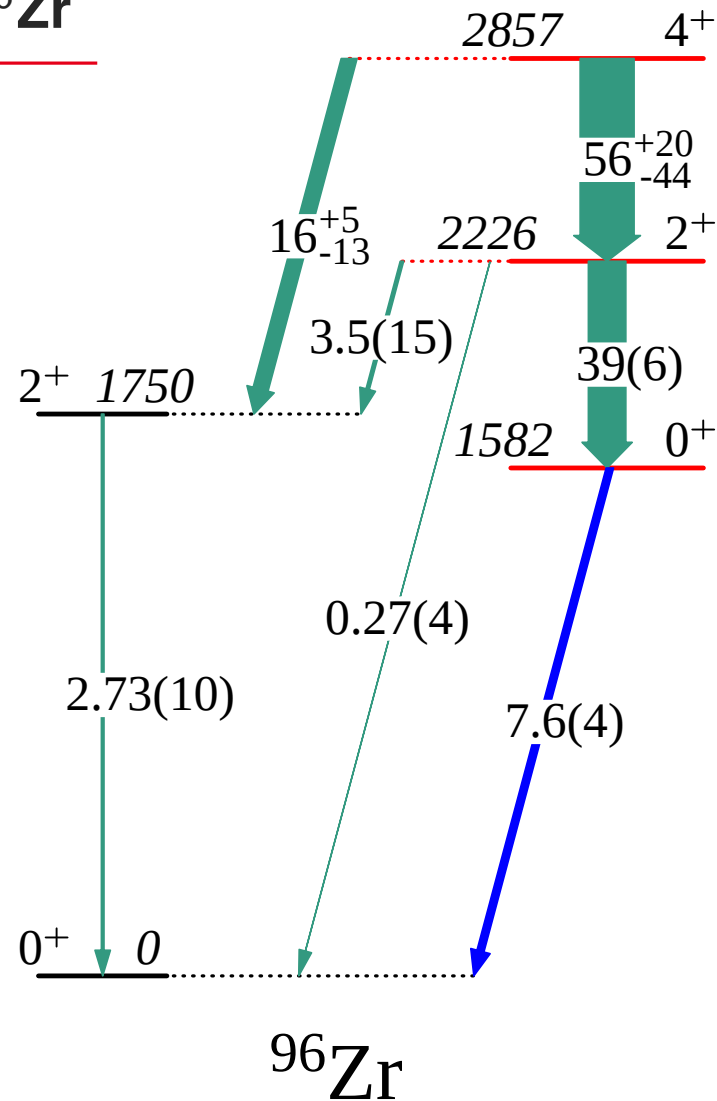
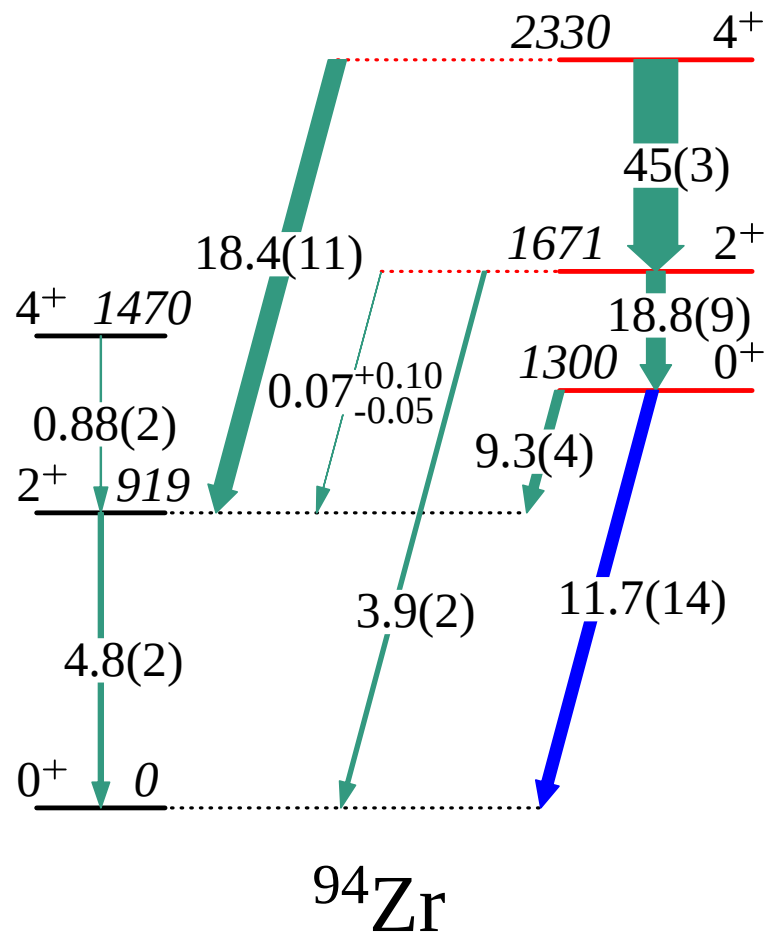
Complementary β -decay measurement into ^{96}Zr using GRIFFIN at TRIUMF-ISAC

- Implantation of $^{96}\text{Rb}/^{96}\text{Sr}$ beam onto a moving tape
- Decay of 0^- ground state of ^{96}Y to ^{96}Zr (spin 0,1,2 states populated)

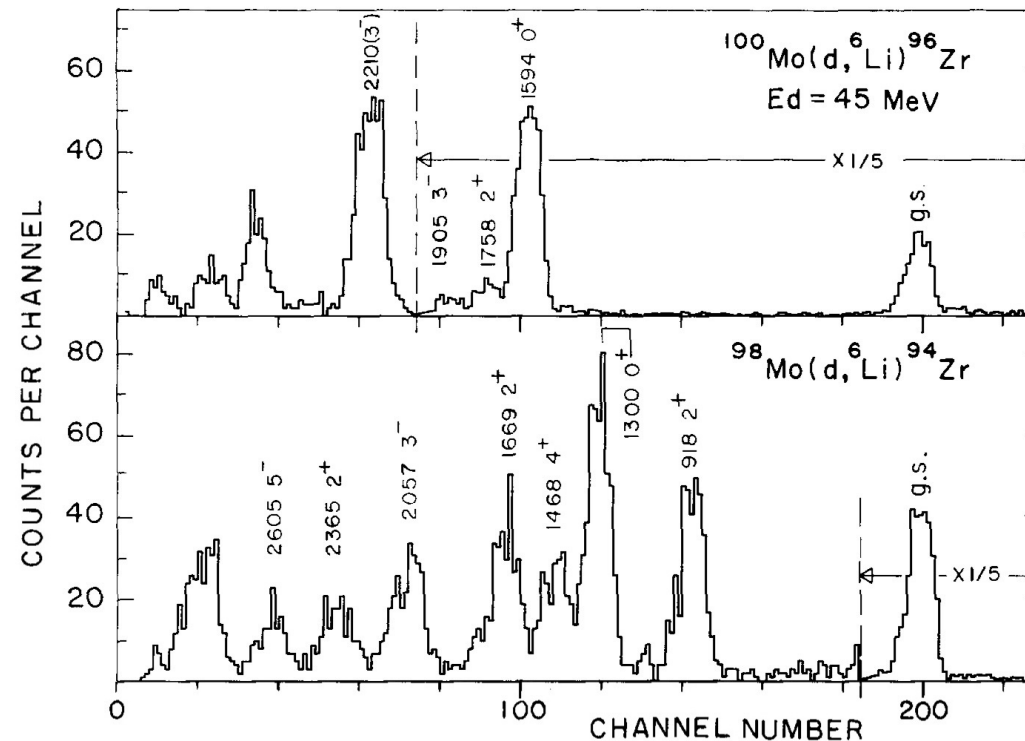


- Significant change in branching ratio of the $2_2^+ \rightarrow 2_1^+$ transition (70.1(15)% vs 57(1)%)
- other branching and mixing ratios in the 2_2^+ decay confirmed (improved precision)
- $B(E2; 2_2^+ \rightarrow 0_2^+) = 39(6) \text{ W.u.}$

Deformed structures in ^{94}Zr and ^{96}Zr



- We observe a significant increase of in-band E2 strength in the deformed structure when moving from ^{94}Zr to ^{96}Zr
- Can we say if they correspond to similar configurations (like in the IBM-CM description)?



- Results of α transfer suggest a very different behaviour of the 0_2^+ states in $^{94,96}\text{Zr}$:
 - ^{94}Zr (and lighter Zr nuclei): population of the 0_2^+ state **5x weaker** than that of the ground state
 - ^{96}Zr : population of the 0_2^+ state **2x stronger** than that of the ground state

Summary

- we pursue a broad experimental programme to study quadrupole and octupole collectivity in Zr nuclei combining Coulomb excitation and β decay:
 - our data show that a weakly-deformed ground state coexists with a more rigid, triaxial-deformed 0_2^+ state in ^{94}Zr
 - the precision on the in-band E2 strength in the deformed structure in ^{96}Zr has been significantly improved
 - in combination with information from α transfer reactions, these data suggest that three different shapes are present in $^{94,96}\text{Zr}$: nearly spherical, deformed triaxial-oblate, and more strongly deformed.
 - what happens in lighter and heavier Zr nuclei? Some answers in the next talk!

