



**32nd
Nuclear Physics
Workshop**

28-30 September 2026
Lublin, Poland



復旦大學
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Nucleon Short-Range Correlations and Equation of State of Dense Matter

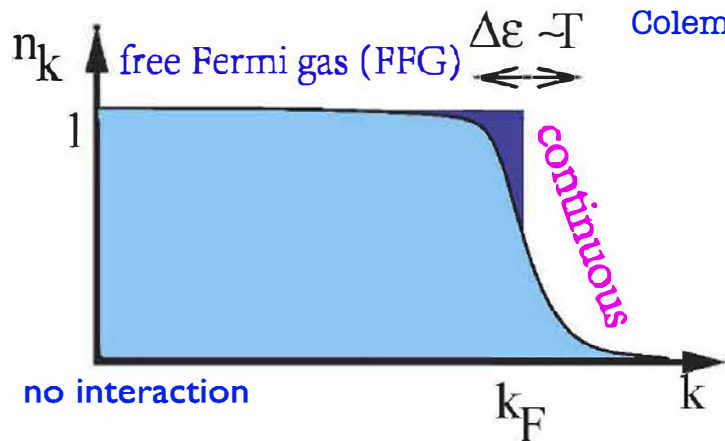
Bao-Jun Cai

Fudan University, 9/29/2026

Collaborators: Lie-Wen Chen (SJTU), Bao-An Li (East Texas A&M), Yu-Gang Ma (Fudan&ECNU)

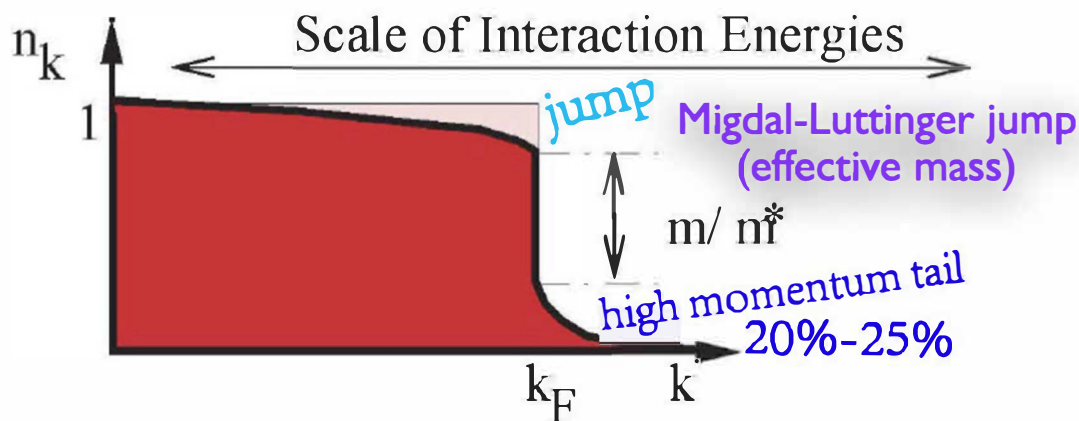
ref.: arXiv:2512.04206

The $n(k)$ at $T=0$



Coleman, Many-body Physics, 2015

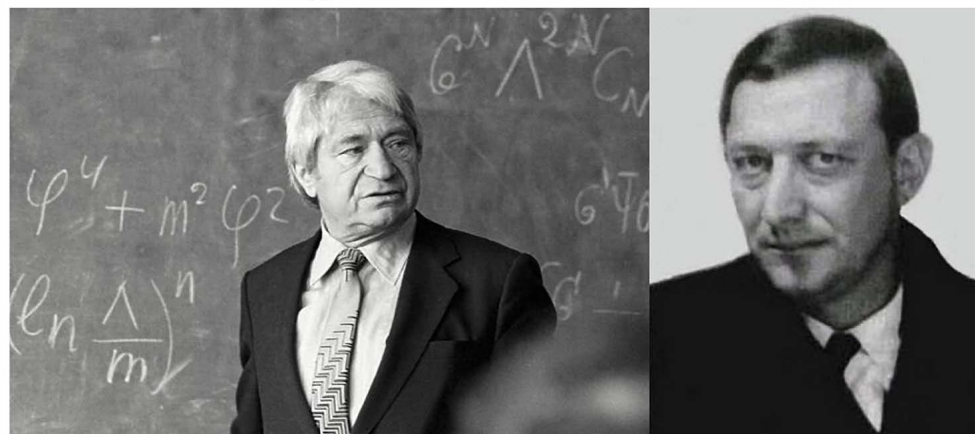
Migdal, JETP, 1957; Luttinger, PR, 1960



$$\text{Fermi-Dirac : } \frac{1}{\exp[(\mu - E_k)/k_B T] + 1}$$

(thermal effect : $T \neq 0$)

even at $T=0$, there is a high momentum tail (HMT) in the single nucleon momentum distribution function $n(k)$



Short-range correlations (SRCs): ~ 1 fm

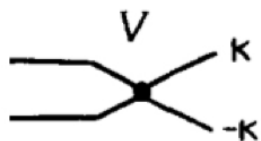
two-body density : $\rho(\mathbf{x}, \mathbf{x}') = [1 + \overbrace{F(\mathbf{x}, \mathbf{x}')^{\text{correlation}}}] \rho(\mathbf{x}) \rho(\mathbf{x}')$
 (probability of finding one nucleon at i and another at i')

↓ Fourier transform

$$n_{\mathbf{k}} = \int \rho(\mathbf{x}, \mathbf{x}') e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')} d\mathbf{x} d\mathbf{x}'$$

Antonov, Hodgson and Petkov, Nucleon Correlations in Nuclei, 1993

pair $\sim \psi \cdot \psi$

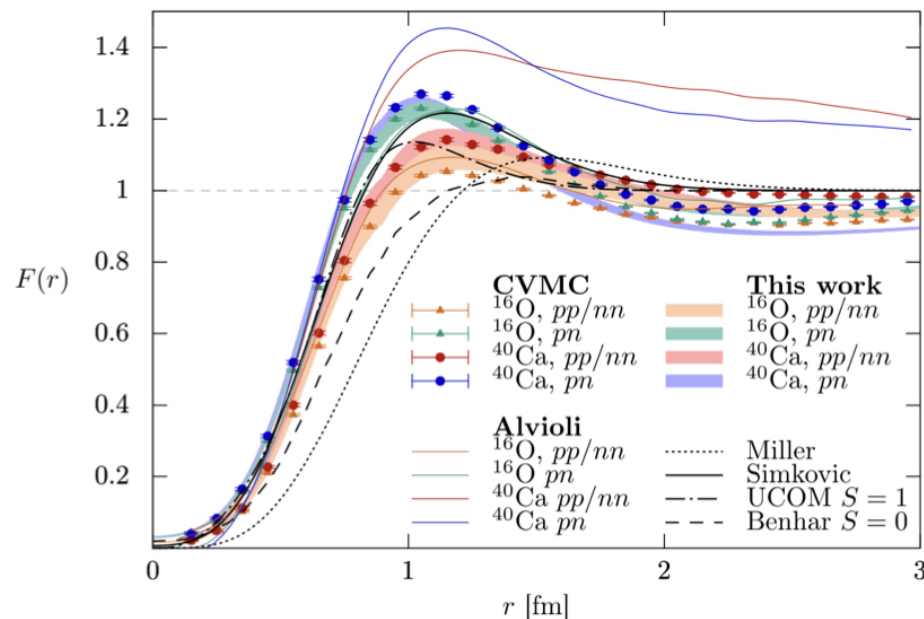


$$n_{\mathbf{k}} \sim \psi^2(k) \xrightarrow{|\mathbf{k}| \rightarrow \infty} \frac{\bar{V}^2(k)}{|\mathbf{k}|^4} \xrightarrow[V(\mathbf{x}) \sim \delta(\mathbf{x}) \rightarrow \bar{V}(k) = \text{const.}]{\text{contact interaction}} \sim \frac{1}{|\mathbf{k}|^4}$$

leading-order term
in pion-less EFT

Amado, PRC, 1976
 Hammer et al., RMP, 2020

Cruz-Torres et al., PLB, 2018



correlation occurs at ~ 1 fm
 $\leftrightarrow$ short-range correlations (SRCs)

Tensor force and OPEP

about 4% high-momentum (HM) component in
deuteron wave function $\leftrightarrow$ d-wave ($L=2$)

(i) the presence of a large high-momentum component of the deuteron wave function:

$$\int \psi_D^2(k) \theta(k - 0.3 \text{ GeV}/c) d^3k \sim (3.5-5)\% ;$$

(ii) dominance of the D-wave in the high-momentum component:

$$\int w^2(k) \theta(k - 0.3 \text{ GeV}/c) d^3k \sim (3-4)\% ; \text{ d-wave component denominates at large momentum } k$$

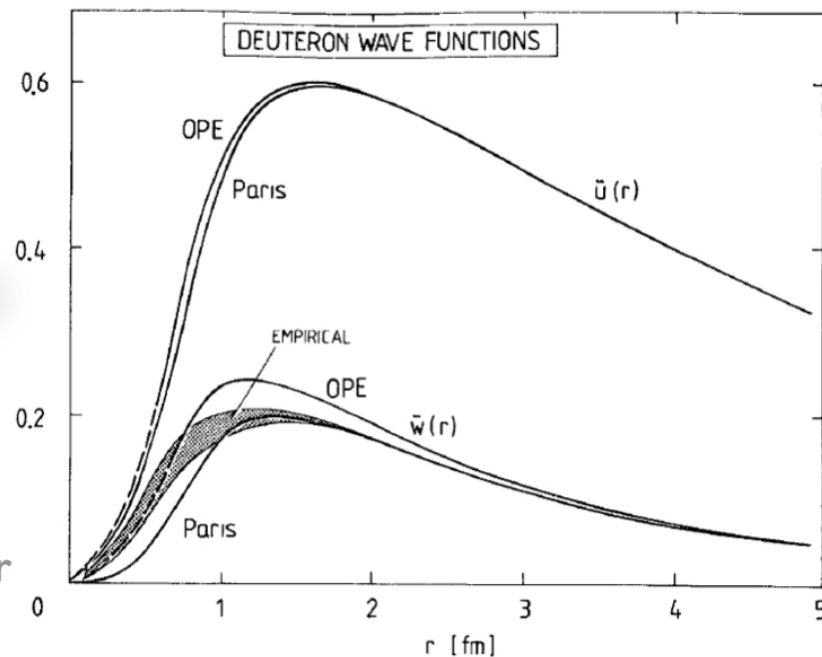
Frankfurt and Strikman, Phys. Rep., 1988

$$V_{\text{OPT}}(\mathbf{x}) = \frac{1}{3} \frac{f^2}{4\pi} \left[\frac{e^{-m_\pi r}}{r} - \frac{4\pi}{m_\pi^2} \delta(\mathbf{x}) \right] (\vec{\sigma}_1 \cdot \vec{\sigma}_2) (\vec{\tau}_1 \cdot \vec{\tau}_2) \\ + \frac{1}{3} \frac{f^2}{4\pi} \left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2} \right) \frac{e^{-m_\pi r}}{r} \text{tensor operator } S_{12} (\vec{\tau}_1 \cdot \vec{\tau}_2)$$

small r :

$$V_{\text{OPE}}(\mathbf{x}) \sim \frac{1}{r^3} \leftrightarrow \text{singular} \leftrightarrow \text{manifestations as SRCs}$$

Ericson and Rosa-Clot, ARNPS, 1985
Ericson and Weise, Pion and Nuclei, 1988



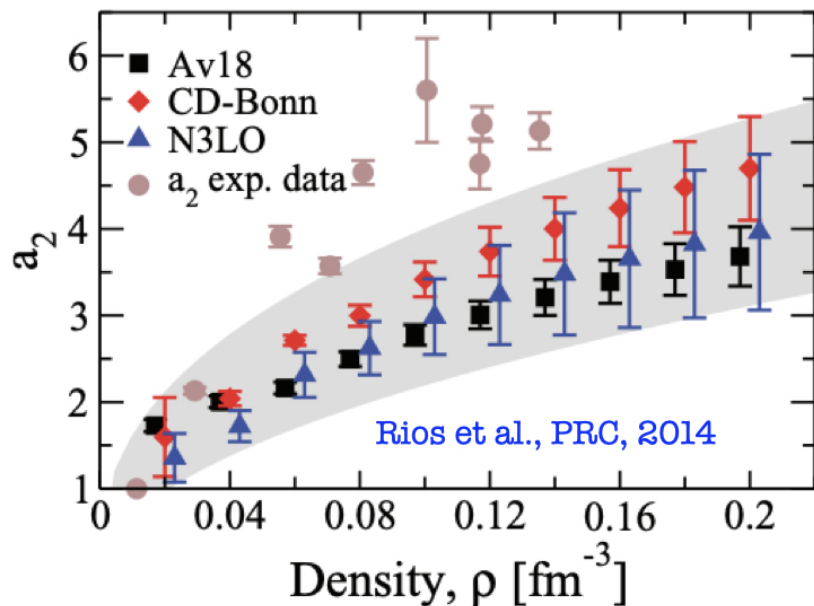
u : s-wave function

w : d-wave function

$\leftrightarrow$ tensor force $\leftrightarrow$ np $\gg$ pp

SRC pairs in nuclei and nuclear matter

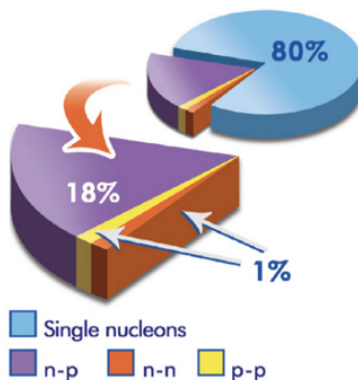
$a_2(A) = n_k^A / n_k^D$: probability of finding a HM pair in nucleus A relative to deuteron



$a_2(\infty) \approx 7 \pm 1$:

SNM: $28\% \pm 4\%$; PNM: 1%-2%

self-consistent Green's
function theories



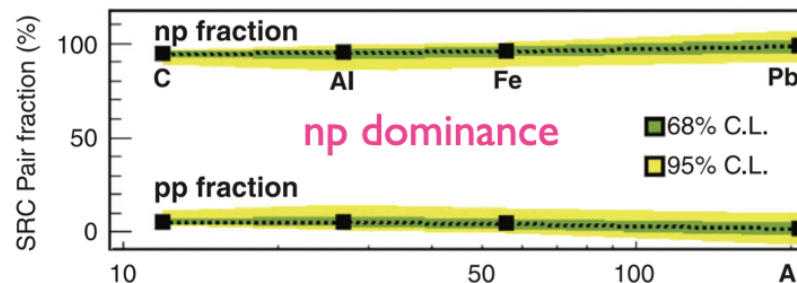
$\phi_0 \lesssim 2.5 \leftrightarrow \phi_0 k_F \lesssim 700 \text{ MeV (exp.)}$

ϕ_0 : high momentum cutoff

C_0 : HMT strength

$$\text{SNM: } n_k^0 = C_0 \left(\frac{k_F}{|k|} \right)^4 \text{ for } k_F \lesssim |k| \lesssim \phi_0 k_F$$

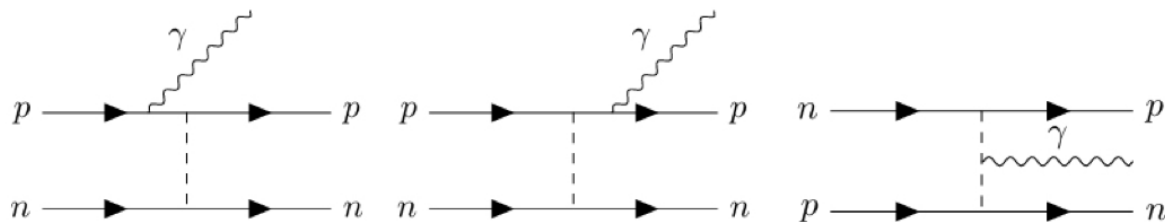
$$x_{\text{SNM}}^{\text{HMT}} = \int_{k_F}^{\phi_0 k_F} n_k^0 dk / \int_0^{\phi_0 k_F} n_k^0 dk = 3C_0 (1 - \phi_0^{-1})$$



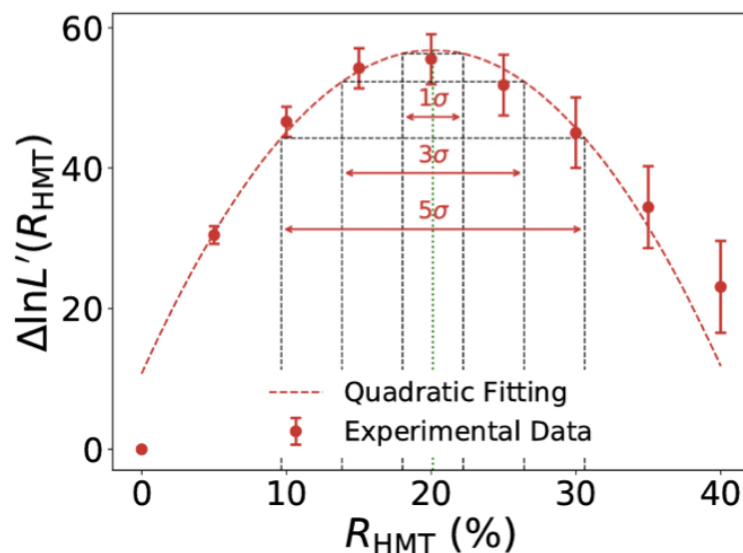
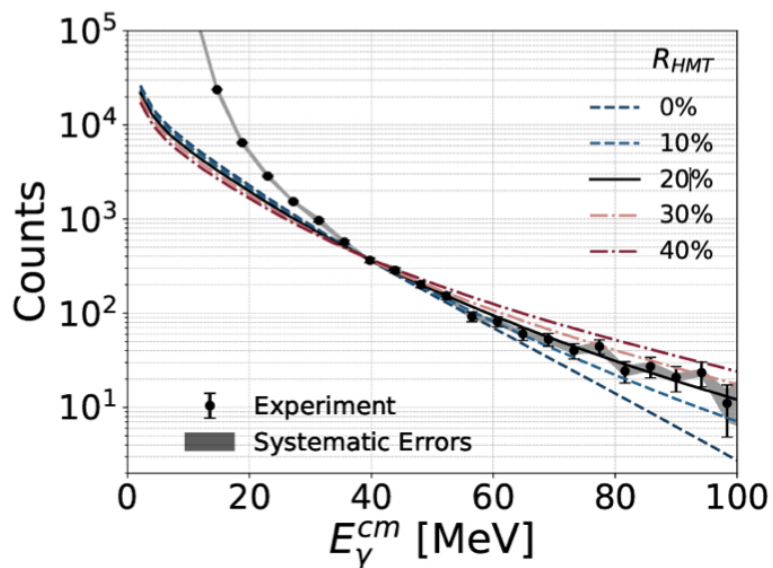
Subedi et al., Science, 2008; Hen et al., Science, 2014

Bremsstrahlung Gamma Ray Emission

Z.G. Xiao Group, Tsinghua University



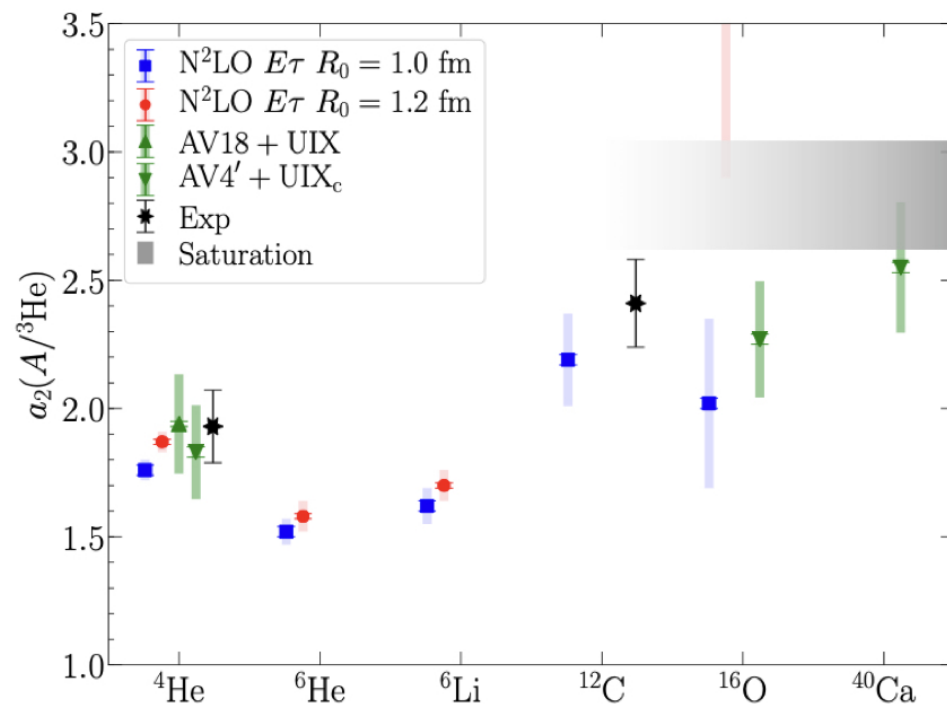
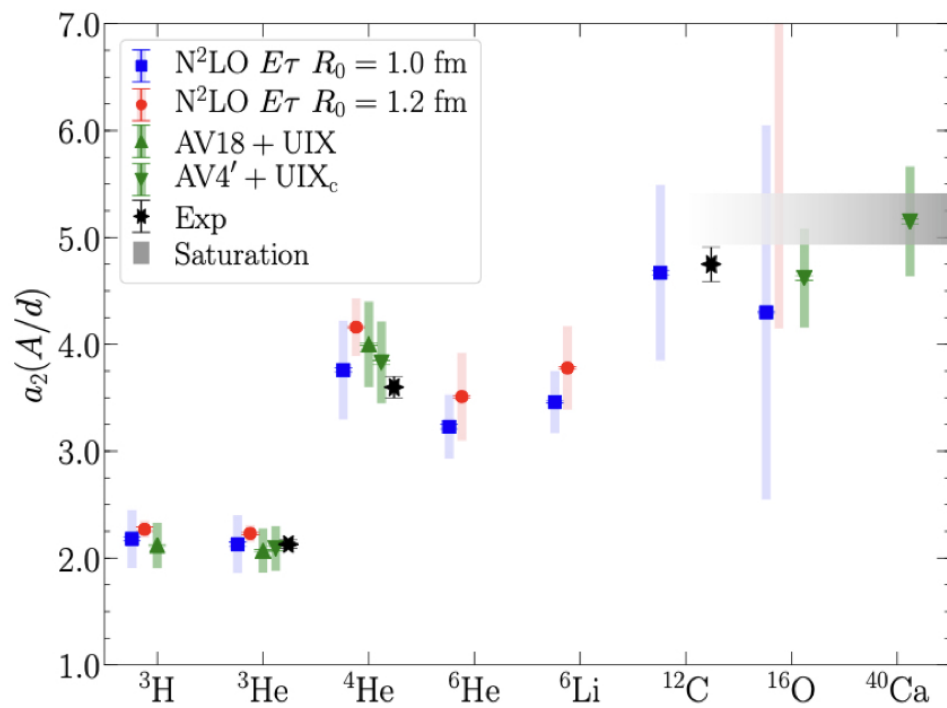
SNM: $\approx 20\%$ in ^{124}Sn
(IBUU-MDI simulation)



Scaling factor from ab initio with chiral EFT

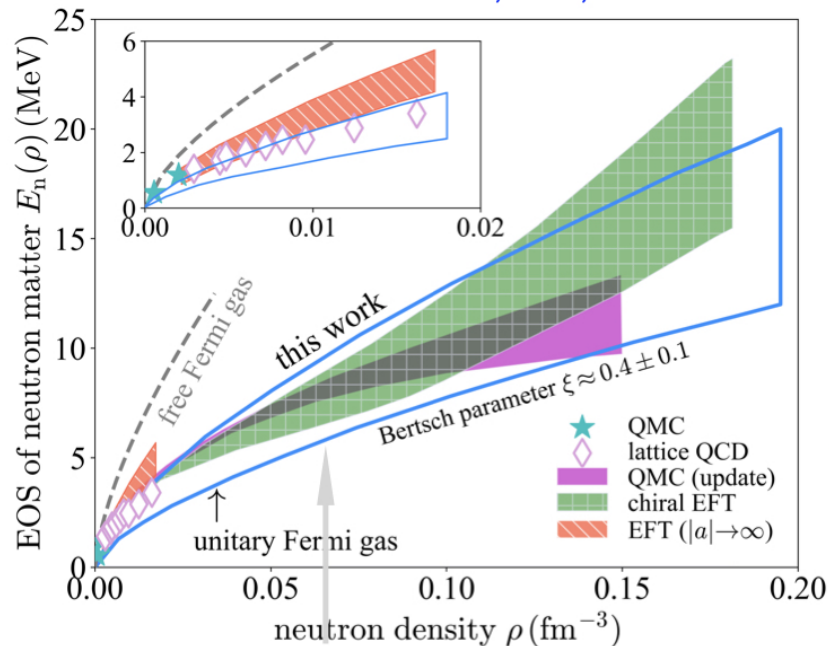
$a_2 \approx 5.2 \pm 0.2$ consistent with Xiao's group result

$x_{\text{SNM}} \approx a_2 \cdot \underbrace{x_{\text{deuteron}}}_{\approx 4\%}$



Pure neutron matter: Tan's contact relations

B.J. Cai and B.A. Li, PRC, 2015



$$\text{PNM: } E_n(\rho) \approx \frac{3k_F^{n,2}}{52M} \left[\xi - \zeta \left(\frac{1}{k_F^n a} \right) - \frac{5\nu}{3} \left(\frac{1}{k_F^n a} \right)^2 \right]$$

$\xi \approx 0.4$: Bertsch parameter; $\zeta \approx \nu \approx 1$: universal

Bertsch, 1999; Bulgac, PRL, 2005

18

Tan Relations and the Contact Parameter

S.N. Tan, Ann. Phys., 2008

unitarity condition: $|a| \gg d \gg r$

$$a(^1S_0) \approx -18.8 \text{ fm}; \quad d \approx \rho_0^{-1/3} \approx 1.8 \text{ fm}; \quad r \approx \frac{1}{2m_\pi} \approx 0.7 \text{ fm}$$

$$\rightarrow C_n k_F^{n,4} = -4\pi M \frac{d(\rho E_n)}{d(a^{-1})} \quad (\text{adiabatic sweep theorem})$$

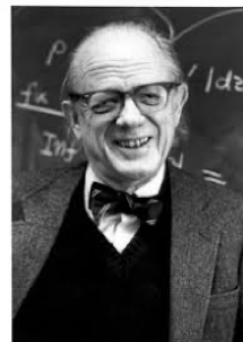
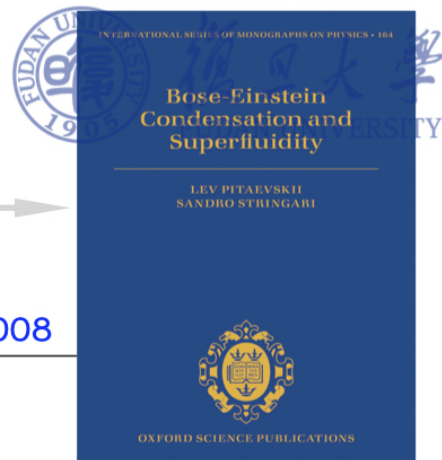
$$C_n (1 - \phi_n^{-1}) = x_{\text{PNM}}^{\text{HMT}} \approx 1\%-2\%$$

$$|a| \gg d \gtrsim r$$

ultracold atoms:

$$a \gg d \gg r$$

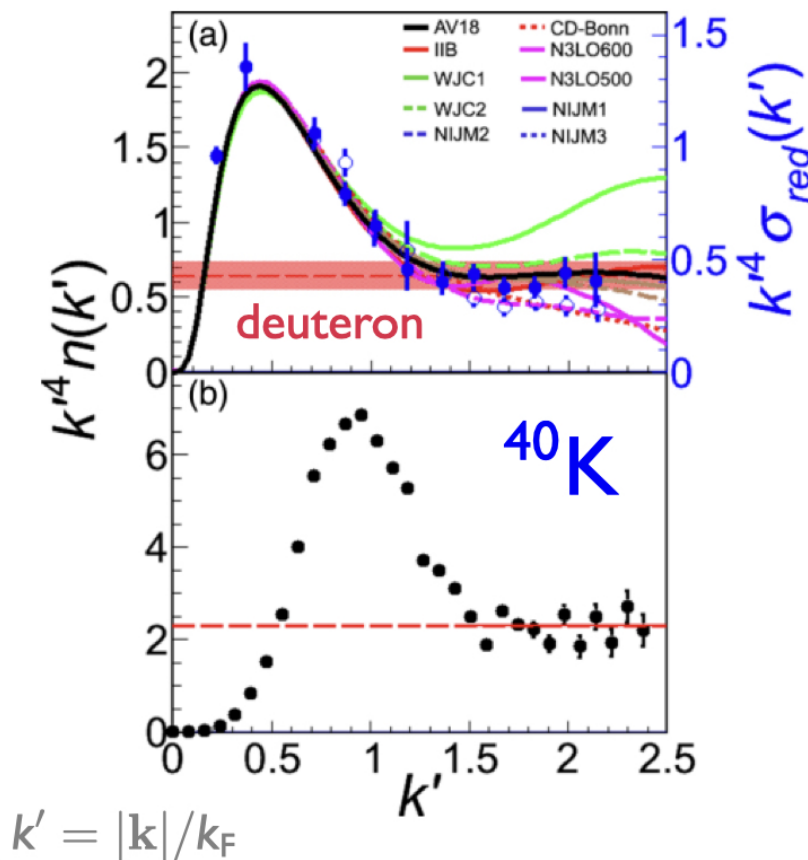
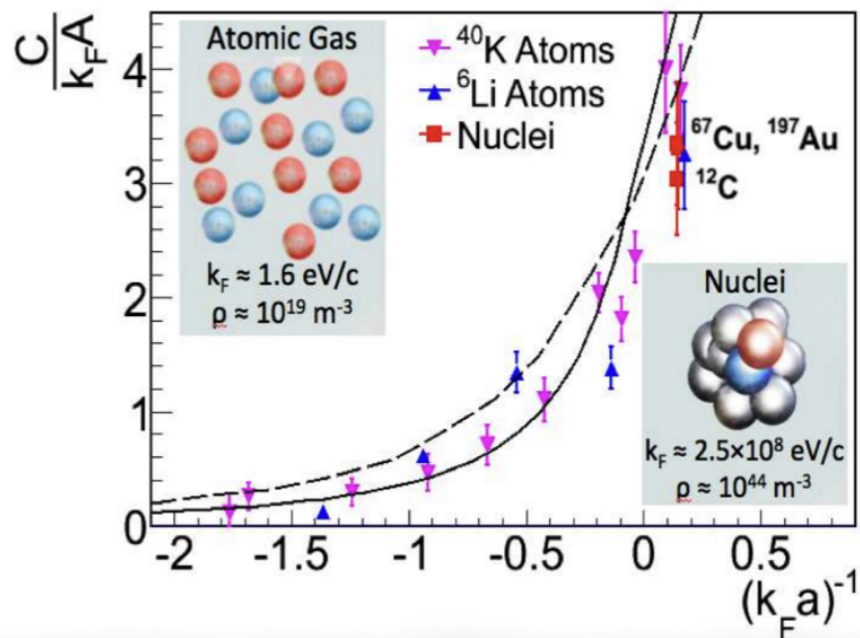
(Feshbach resonance)



Similarity: ultracold Fermi gas & finite nuclei

ultracold Fermi gas and nuclei:

- (1) differs by $\gtrsim 25$ orders of magnitude in density
- (2) Fermion-Fermion interactions are very different



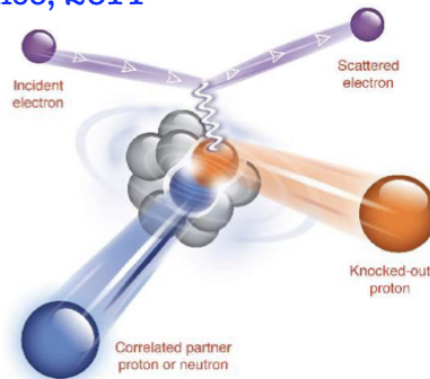
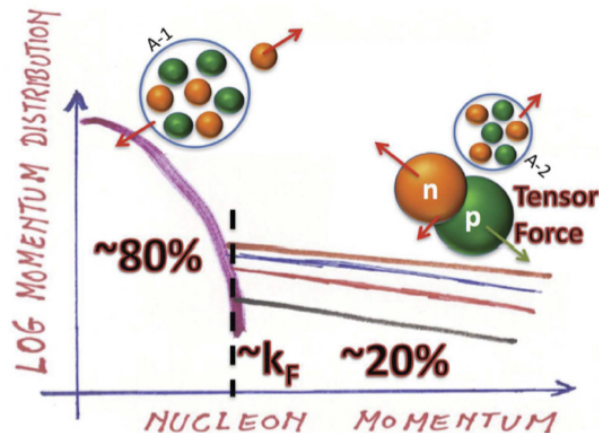
deep quantum nature of these many-body systems

Hen et al., PRC, 2015

Summary: $n(k)$ with (isospin-dependent) HMT



Hen et al., RMP, 2017; Hen et al., Science, 2014



$$n_k^I(\rho, \delta) = \begin{cases} \Delta_J + \beta_J(|\mathbf{k}|/k_F)^2, & 0 < |\mathbf{k}| < k_F \\ C_J(k_F/|\mathbf{k}|)^4, & k_F < |\mathbf{k}| < \phi_J k_F \end{cases}$$

zero-momentum depletion

$$\mathcal{P}_J = \mathcal{P}_0 \left(1 + \tau_3^J \mathcal{P}_J \delta \right)$$

25% HMT: SNM
1%-2% HMT: PNM } → strong isospin dependence

analogy: angle-resolved photoemission spectroscopy (ARPES) → electron wf

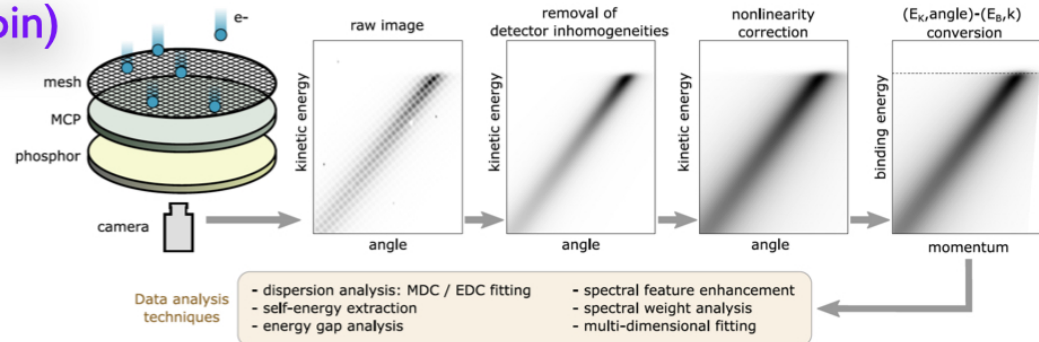
nuclear interaction is more complicated ($\rho, \delta, k, \text{spin}$)

spectral function: $\mathcal{A}(\mathbf{k}, \omega) = 2\text{Im} G(\mathbf{k} - i0^+) \sim d\sigma$

hole

$$\mathcal{A}(\mathbf{k}, \omega)/2\pi = \underbrace{Z_{\mathbf{k}} \delta(\omega - \epsilon_{\mathbf{k}})}_{\epsilon_{\mathbf{k}} = k^2/2M - \mu} + \text{continuum}$$

$$n_{\mathbf{k}} = \int_{-\infty}^0 \frac{d\omega}{2\pi} \mathcal{A}(\mathbf{k}, \omega) = Z_{\mathbf{k}} \Theta(-\epsilon_{\mathbf{k}}) + \frac{C}{|\mathbf{k}|^4} \Theta(\epsilon_{\mathbf{k}})$$



Early derivations on $n(k)$ with a k^{-4} HMT



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THE MOMENTUM DISTRIBUTION IN A DILUTE FERMI GAS AT ZERO TEMPERATURE

W. CZYŻ[†] and K. GOTTFRIED^{††}

Institute for Theoretical Physics, University of Copenhagen, Denmark

Received 2 September 1960

According to (11), $n(p)$ varies as p^{-4} when $(p/p_F) \rightarrow \infty$. The expressions given above are not, however, expected to be valid when $pa > 1$, since the detailed structure of the interparticle force, and not just the S-wave scattering length, comes into play for such large momenta. Therefore the p^{-4} -dependence should only be taken seriously when the inequalities $p \gg p_F$ and $pa < 1$ are satisfied simultaneously.



new features:

- (1) sizable fraction of HMT nucleons: $\sim 25\%$ (SNM)
- (2) isospin dependence of the HMT (SNM+PNM)
- (3) without the restriction of smallness: $k_F^* a \ll 1$
- (4) experimental data available (e.g., HIAF, JLab, ...)

Czyz and Kottfried, Nucl. Phys., 1960

Belyakov, JETP, 1961; Sartor and Mahaux, PRC, 1980, ...

THE MOMENTUM DISTRIBUTION OF PARTICLES IN A DILUTE FERMI GAS

V. A. BELYAKOV

Moscow Engineering Physics Institute

Submitted to JETP editor November 26, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 1210-1212 (April, 1961)

The first two terms of the expansion in powers of the gas parameter of the momentum distribution of particles in a non-ideal Fermi gas are obtained, using a perturbation method.

$$x = |k|/k_F$$

We quote the values of the function $n(p)$ at several points:

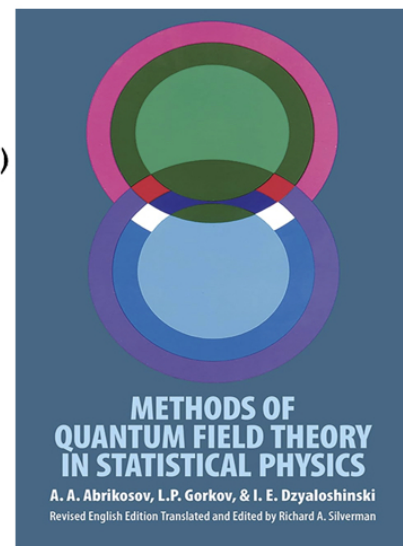
$$n(0) = 1 - 2 (3na^3/\pi)^{2/3} \left(1 - \frac{1}{2} \ln 2\right),$$

$$n(p_0 - 0) = 1 - 2 (3na^3/\pi)^{2/3} \left(\frac{1}{3} + \ln 2\right),$$

$$n(p_0 + 0) = 2 (3na^3/\pi)^{2/3} \left(\ln 2 - \frac{1}{3}\right),$$

$$n(p) = (4/9x^4) (3na^3/\pi)^{2/3} \text{ as } p \rightarrow \infty.$$

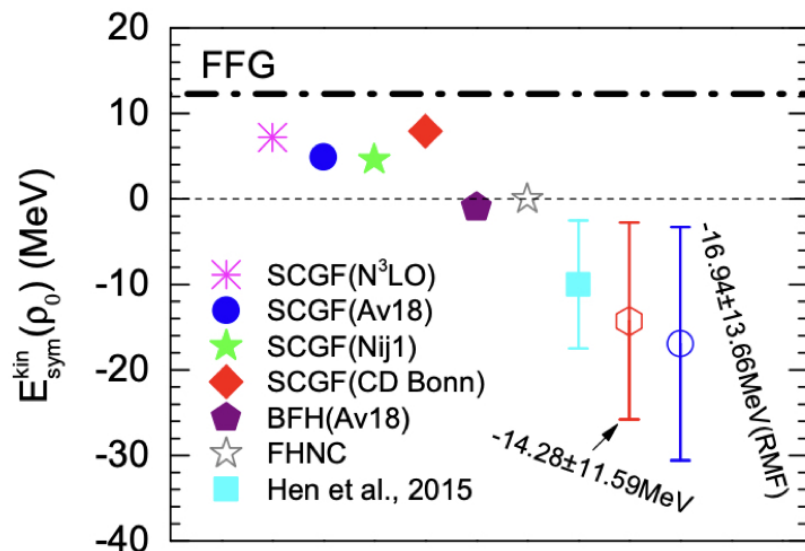
$$|k_F a| \ll 1$$



Kinetic EOS with SRC-HMT

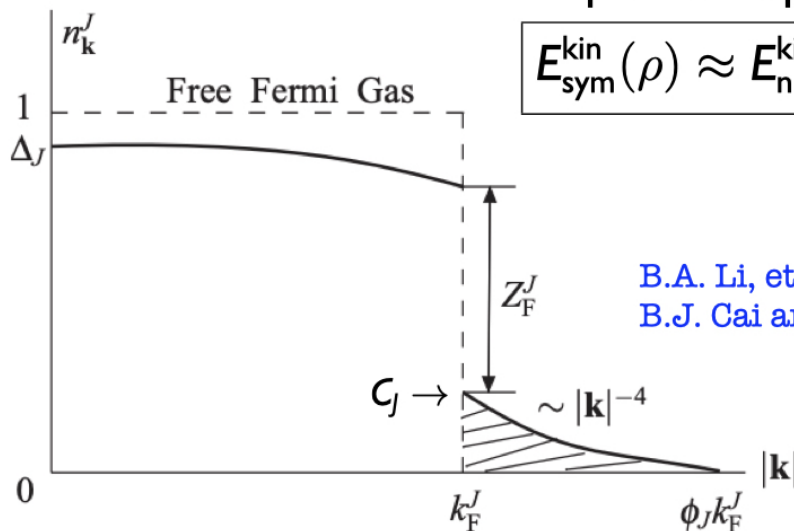
kinetic energy per nucleon (kinetic EOS):

$$E^{\text{kin}}(\rho, \delta) = \frac{1}{\rho} \underbrace{\frac{2}{(2\pi)^3} \sum_{j=n,p} \int_0^\infty \frac{k^2}{2M} n_k^j(\rho, \delta) dk}_{\text{kinetic energy density } \varepsilon^{\text{kin}}(\rho, \delta)}$$



parabolic approximation

$$E_{\text{sym}}^{\text{kin}}(\rho) \approx E_n^{\text{kin}}(\rho) - E_0^{\text{kin}}(\rho)$$



B.A. Li, et al., PPNP, 2018
B.J. Cai and B.A. Li, PRC, 2016

- (1) about 28% np at HM state in SNM $\rightarrow E_0(\rho)$ is enhanced
- (2) 1%-2% neutrons at HM state in PNM $\rightarrow E_n(\rho) \sim$ free Fermi gas
- (3) symmetry energy $\approx$ PNM-SNM (parabolic approximation)

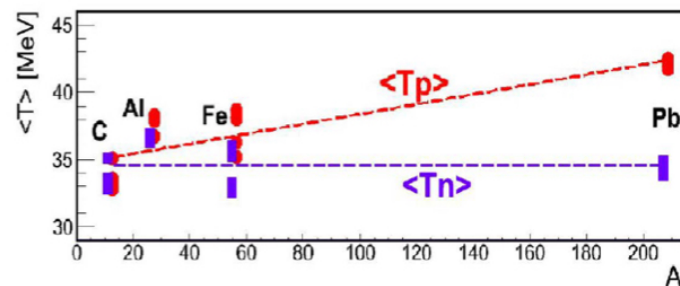
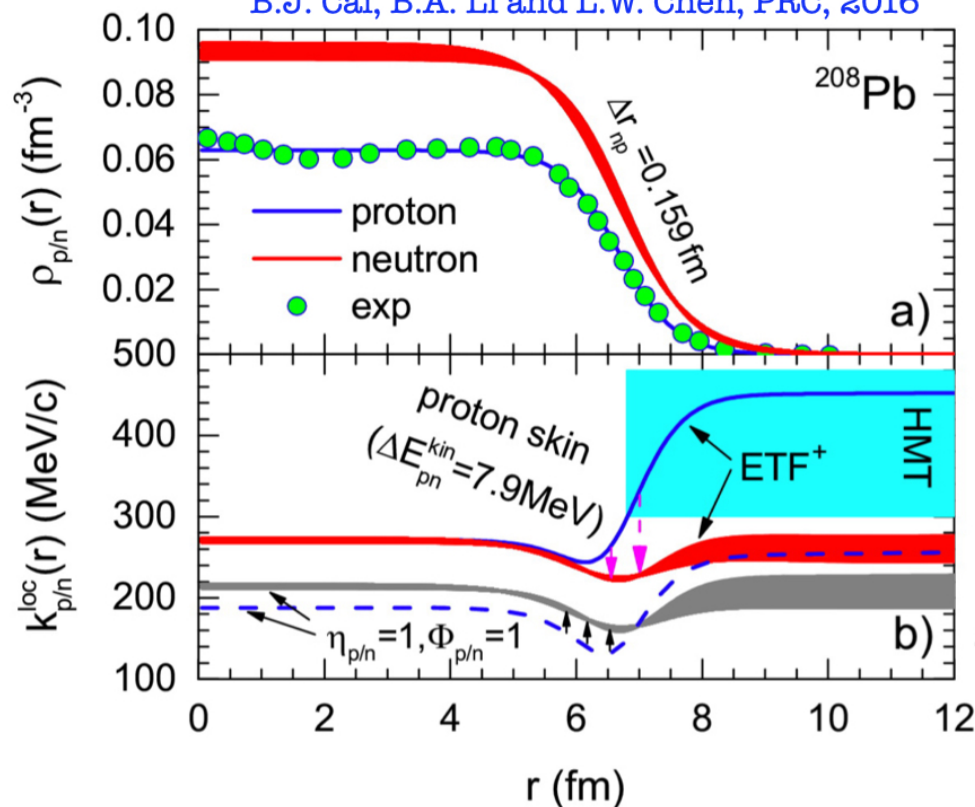
Where these high-momentum protons live?



protons move faster than neutrons in neutron skins

Hen et al., Science, 2014

B.J. Cai, B.A. Li and L.W. Chen, PRC, 2016



extended Thomas-Fermi (TF) approximation (ETF⁺):

$$\varepsilon_j^{\text{kin}}(r) = \frac{1}{2M} [\tau_{\text{TF}}[\rho_j] + \tau_2[\rho_j] + \dots]$$

$$\text{TF term: } \tau_{\text{TF}}[\rho_j] = \frac{2}{(2\pi)^3} \int_0^{\phi_j k_F^j} \mathbf{k}^2 n_{\mathbf{k}}^j d\mathbf{k} = \frac{3}{5} (3\pi^2)^{2/3} \rho_j^{5/3} \Phi_j$$

$$\Phi_j = 1 + C_j (5\phi_j + 3/\phi_j - 8) \leftrightarrow \text{HMT factor}$$

surface contribution

$$\text{Weizsäcker term: } \tau_2[\rho_j] = \frac{\eta_j}{36} \frac{(\nabla \rho_j(r))^2}{\rho_j(r)} + \frac{1}{3} \Delta \rho_j(r)$$

ρ_j : 2-parameter Fermi distribution

$$\text{local momentum: } \varepsilon_j^{\text{kin}}(r)/\rho_j(r) = [k_j^{\text{loc}}(r)]^2/2M$$

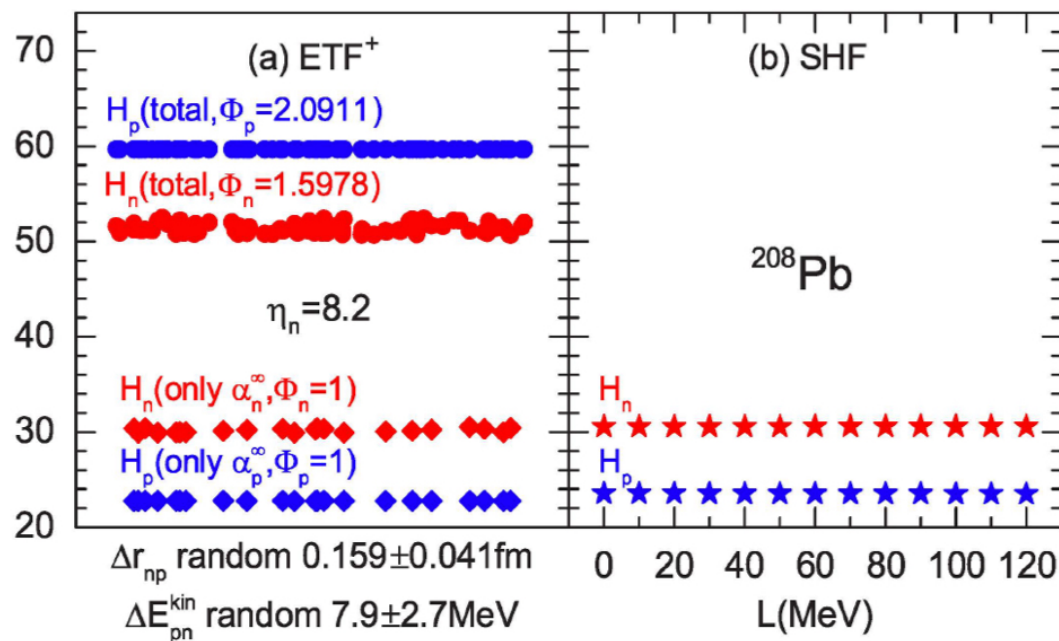
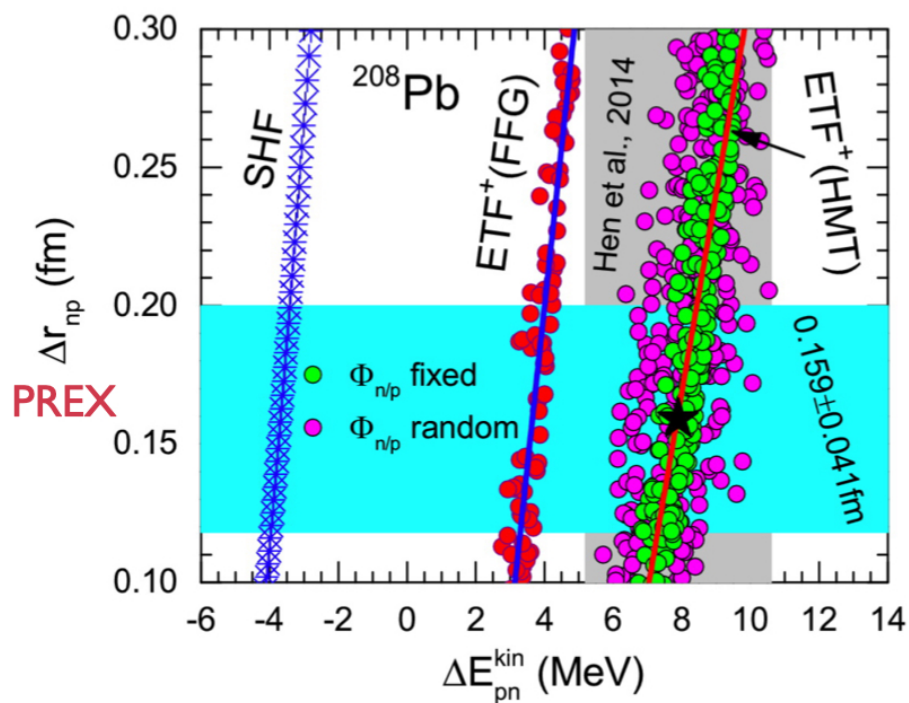
Correlation between neutron-skin and proton-skin

neutron skin: $\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$

proton skin: $\Delta E_{pn}^{\text{kin}} \equiv \langle T_p \rangle - \langle T_n \rangle$

$$\left(\langle T_J \rangle = \int \varepsilon_J^{\text{kin}}(r) dx / \int \rho_J(r) dx \right)$$

$$H_J \equiv \langle r_J^2 \rangle \langle k_J^2 \rangle = \underbrace{\langle r_J \rangle^2 \langle k_J \rangle^2}_{\approx \text{const.} \leftrightarrow \text{Liouville theorem}} \left[1 + \underbrace{\left(\frac{\delta r_J}{\langle r_J \rangle} \right)^2 + \left(\frac{\delta k_J}{\langle k_J \rangle} \right)^2}_{\text{small relative fluctuations}} \right] + \underbrace{\delta^2 r_J \delta^2 k_J}_{\approx \text{const.} \leftrightarrow \text{Heisenberg uncertainty}} \approx \text{const.}$$



B.J. Cai, B.A. Li and L.W. Chen, PRC, 2016

Need a potential $\longrightarrow$ full EOS of NM

kinetic EOS:

momentum-dependence interaction (MDI)

$$E^{\text{kin}}(\rho, \delta) = \underbrace{\frac{1}{\rho} \frac{2}{(2\pi)^3} \sum_{j=n,p} \int_0^\infty \frac{k^2}{2M} n_k^j(\rho, \delta) dk}_{\text{kinetic energy density } \varepsilon^{\text{kin}}(\rho, \delta)}$$

see, e.g., on MDI:
Gale et al., PRC, 1987
B.A. Li, PRC, 2004

potential EOS:

$\Omega(\mathbf{k}, \mathbf{k}')$: momentum-dependence kernel

$$E_{\text{mom}}^{\text{pot}}(\rho, \delta) = \sum_{jj'} \frac{C_{jj'}}{\rho \rho_0} \int d\mathbf{k} d\mathbf{k}' f_j(\mathbf{x}, \mathbf{k}) f_{j'}(\mathbf{x}, \mathbf{k}') \Omega(\mathbf{k}, \mathbf{k}')$$

$$E_{\text{den}}^{\text{pot}}(\rho, \delta) = \frac{A_l(\rho_p^2 + \rho_n^2)}{2\rho\rho_0} + \frac{A_u\rho_p\rho_n}{\rho\rho_0} + \frac{B}{\sigma + 1} \left(\frac{\rho}{\rho_0} \right)^\sigma (1 - x\sigma^2)$$

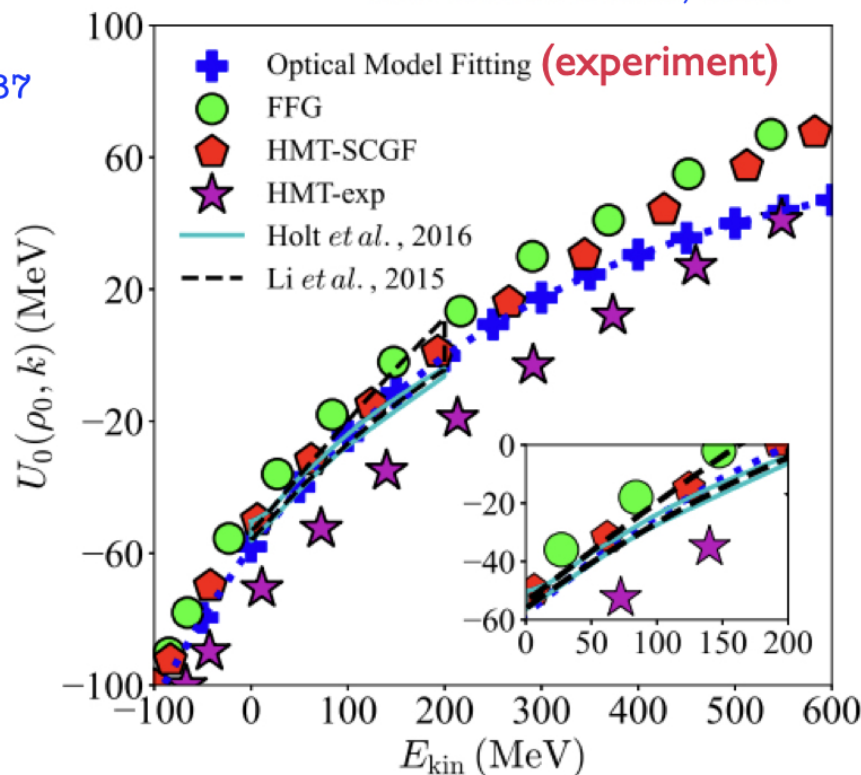
full EOS:

x : 3-nucleon force

$$E(\rho, \delta) = E^{\text{kin}}(\rho, \delta) + E_{\text{mom}}^{\text{pot}}(\rho, \delta) + E_{\text{den}}^{\text{pot}}(\rho, \delta)$$

phase-space distribution: $f_j(\mathbf{x}, \mathbf{k}) = \frac{1}{4\pi^3} n_k^j(\rho, \delta)$

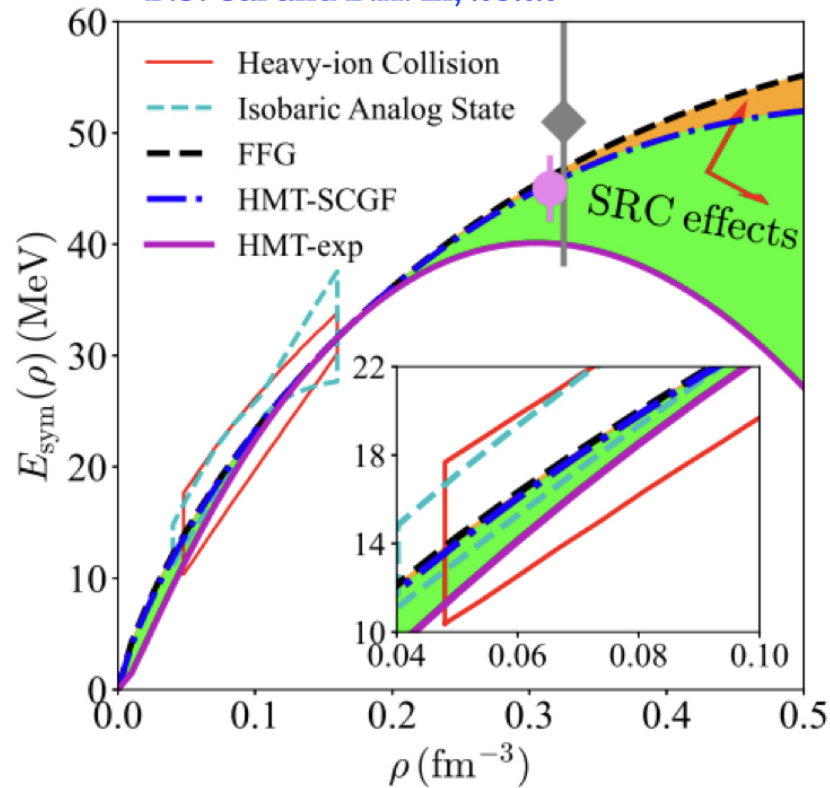
B.J. Cai and B.A. Li, 2022



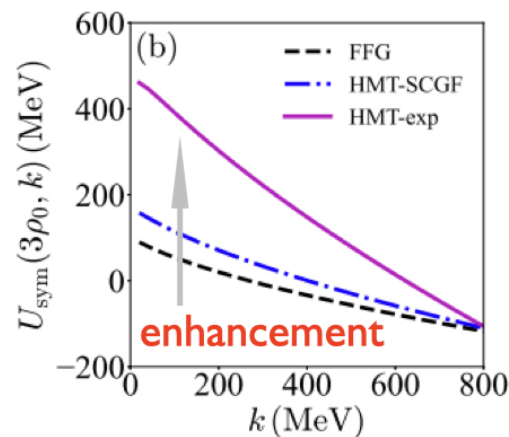
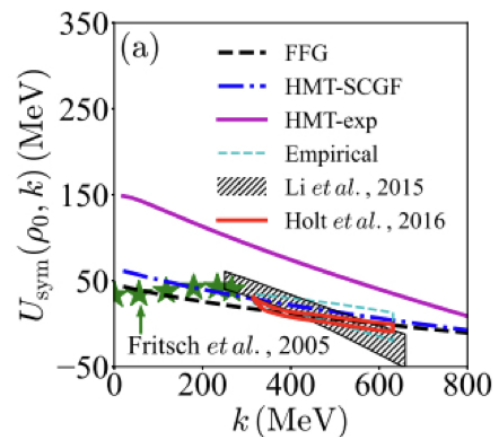
Softening/reduction of total symmetry energy



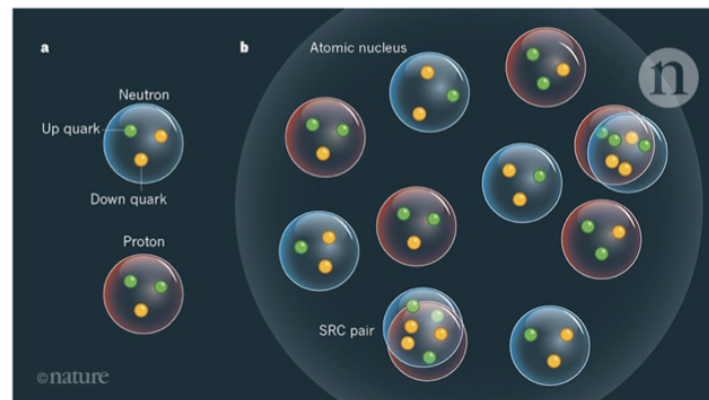
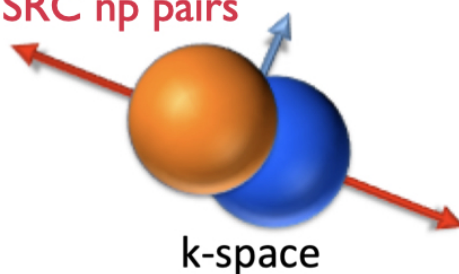
B.J. Cai and B.A. Li, 2022



enhancement due to HMT $\longleftrightarrow$ relevant for HICs



SRC np pairs



SRC nucleon pairs tend to reduce the pressure of the system, therefore softening the symmetry energy

Viewpoint from Hugenholtz-Van Hove Theorem



$$E_{\text{sym}}(\rho) = \frac{k_F^2}{6M_0^*} + \frac{1}{2}U_{\text{sym}}(\rho, k_F) + \overbrace{\Delta E_{\text{sym}}^{\text{HMT}}(\rho)}^{\text{SRC-HMT}}$$

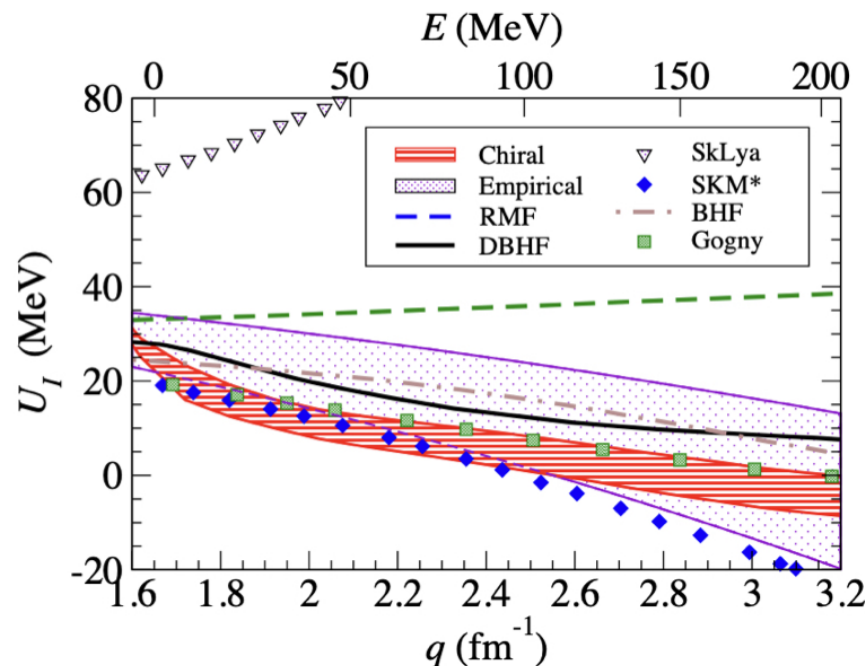
$$M_0^*/M \approx 0.8 \rightarrow k_F^2/6M_0^* \approx 15 \text{ MeV}$$

$$\text{Optical model: } 2^{-1}U_{\text{sym}}(\rho_0, k_F) \approx 15\text{-}16 \text{ MeV}$$

$$E_{\text{sym}}(\rho_0) \approx 30\text{-}31 \text{ MeV} \leftrightarrow \overbrace{\Delta E_{\text{sym}}^{\text{HMT}}(\rho_0)}^{\text{SRC-HMT}} \approx 0 \text{ MeV}$$

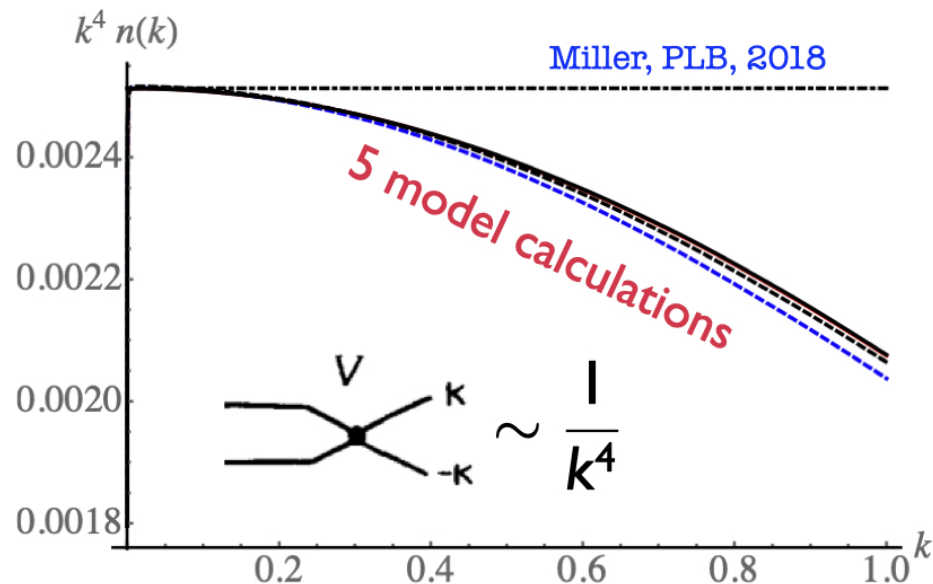
$$L(\rho) = \frac{k_F^2}{3M_0^*} + \left[\frac{k_F^2}{6} \frac{\partial^2 U_0}{\partial k^2} + \frac{k_F}{6} \frac{\partial U_0}{\partial k} + k_F \frac{\partial U_{\text{sym}}}{\partial k} \right]_{k_F} + \frac{3}{2}U_{\text{sym}}(\rho, k_F) + 3U_{\text{sym},2}(\rho, k_F) + \underbrace{\Delta L^{\text{HMT}}(\rho)}_{\text{SRC-HMT}}$$

$$\underbrace{\Delta L^{\text{HMT}}(\rho_0)}_{\text{SRC-HMT}} \neq 0 \text{ MeV} \rightarrow L(\rho_0) \approx 27 \pm 25 \text{ MeV}$$

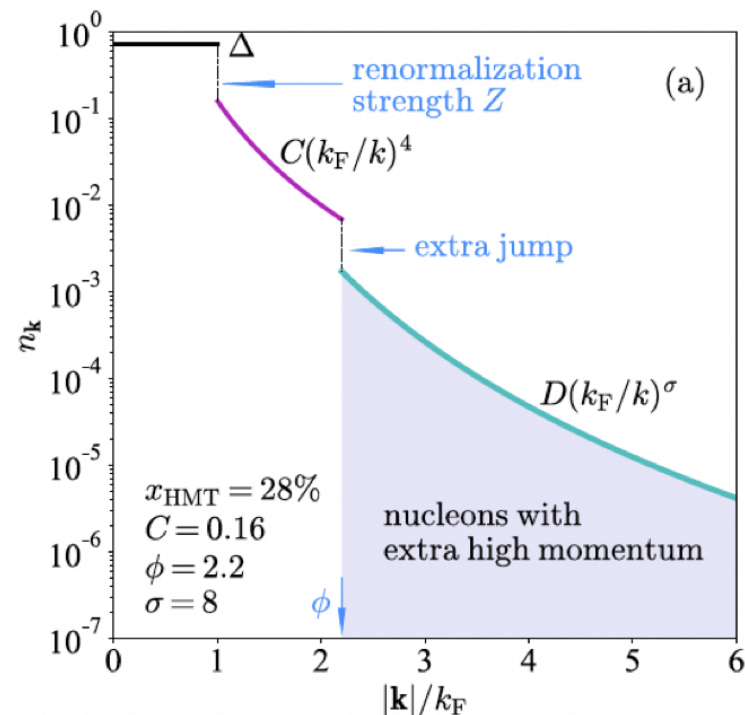


Holt, Kaiser, and Miller, PRC, 2016

HMT in the form $k^{-\sigma}$ with $\sigma \geq 4$



$$n_k = \begin{cases} \Delta, & k \leq k_F, \\ C(k_F/k)^4, & k_F \leq k \leq \phi k_F, \\ D(k_F/k)^\sigma, & k \geq \phi k_F. \end{cases}$$



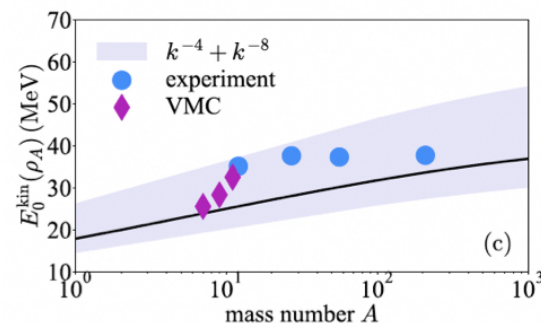
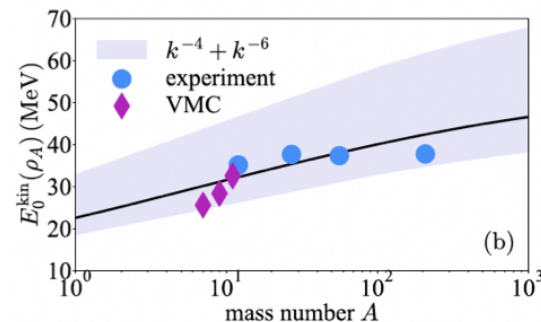
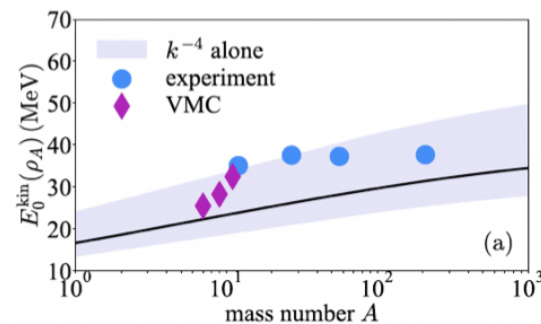
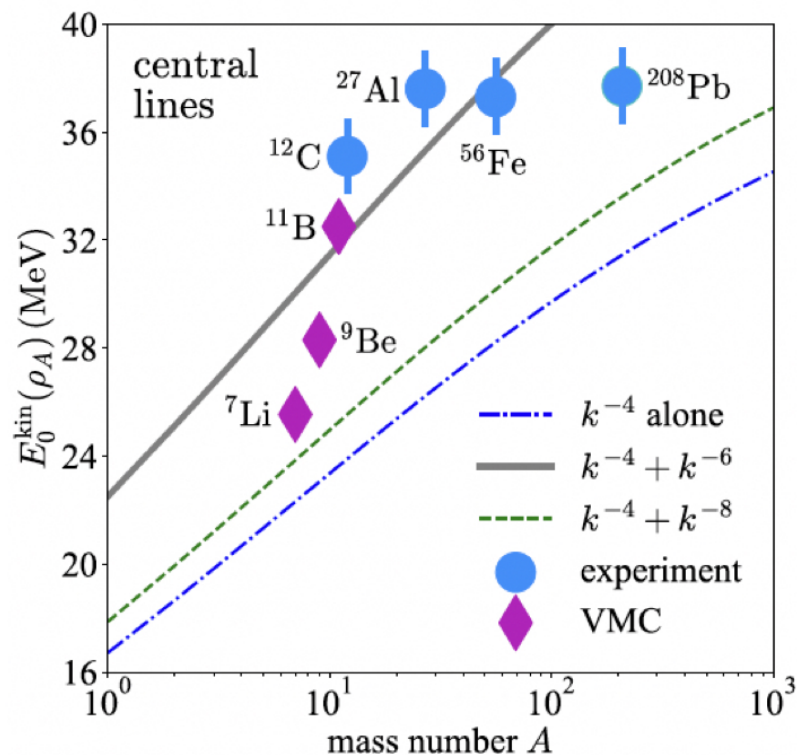
(1) HMT in the form of $k^{-\sigma}$ with $\sigma \geq 4$ exists

(2) after k^{-4} term, next contributions are $k^{-6}, k^{-8}, \dots$

Correction to the kinetic EOS of SNM



$$k^{-\sigma}: \delta E_0^{\text{kin}}(\rho_0) \approx 4.1\text{-}20.2 \text{ MeV}$$



SRC-HMT on neutron star (NS) properties (RMF)

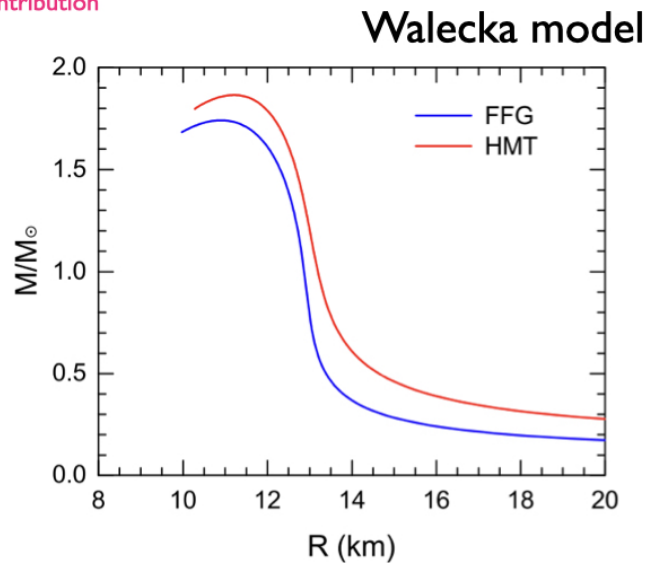
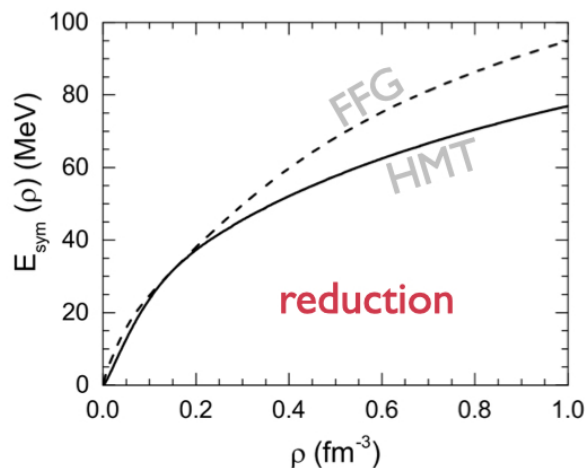
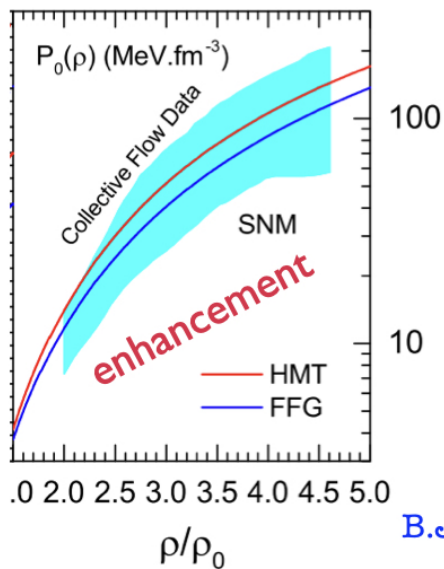


$$\int_0^{k_F^l} \overbrace{s(\mathbf{k}, \rho, \delta) d\mathbf{k}}^{\Theta(k_F^l - |\mathbf{k}|)} \rightarrow \int_0^{\phi_j k_F^l} n_{\mathbf{k}}^l(\text{HMT}) s(\mathbf{k}, \rho, \delta) d\mathbf{k}$$

SRC-HMT acts as a potential mechanism to increase NS mass to be consistent with current observations

e.g., energy density $\varepsilon_j^{\text{kin}}$:

$$\frac{2}{(2\pi)^3} \int_0^{k_F^l} d\mathbf{k} \sqrt{|\mathbf{k}|^2 + M_j^{*,2}} \rightarrow \frac{2}{(2\pi)^3} \left[\int_0^{k_F^l} d\mathbf{k} \Delta_j \sqrt{|\mathbf{k}|^2 + M_j^{*,2}} + \underbrace{\int_{k_F^l}^{\phi_j k_F^l} d\mathbf{k} C_j \left(\frac{k_F^l}{|\mathbf{k}|} \right)^4 \sqrt{|\mathbf{k}|^2 + M_j^{*,2}}}_{\text{HMT contribution}} \right]$$



B.J. Cai and B.A. Li, PRC, 2016

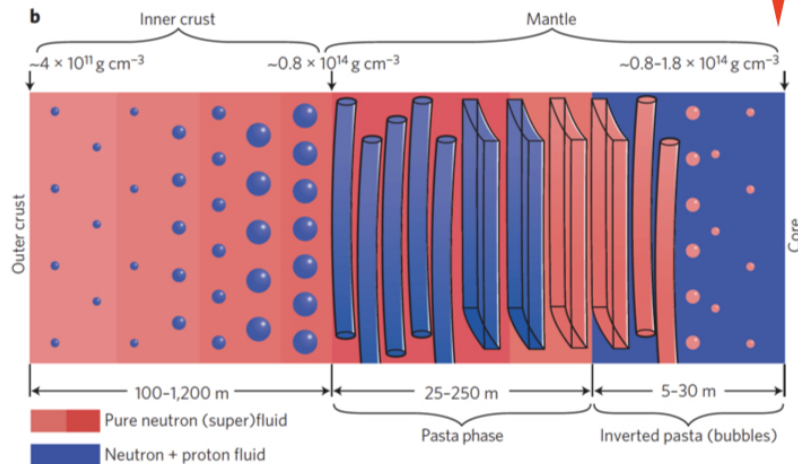
SRC-HMT on low-density properties of NSs



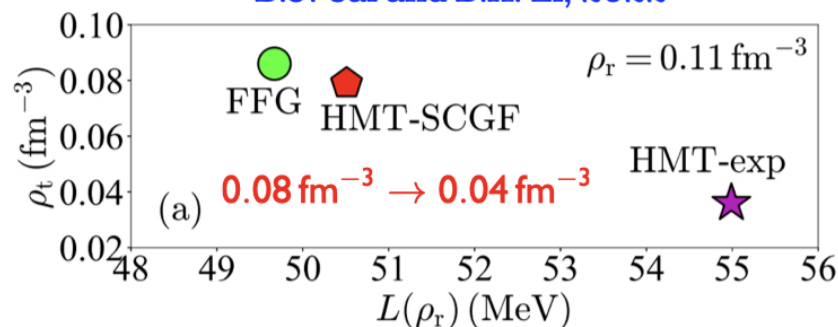
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ρ_t : core-crust transition density in NSs

Newton, Nat. Phys., 2013



B.J. Cai and B.A. Li, 2022



Eur. Phys. J. A (2023) 59:209
<https://doi.org/10.1140/epja/s10050-023-01122-4>

Regular Article - Theoretical Physics

THE EUROPEAN
PHYSICAL JOURNAL A

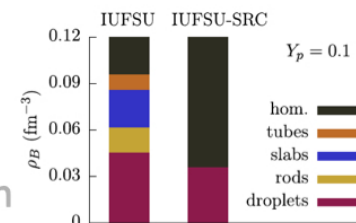
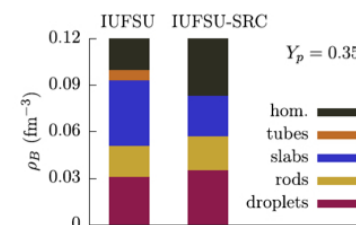
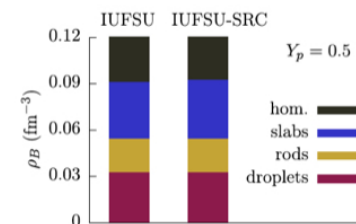


Do short range correlations inhibit the appearance of the nuclear pasta?

M. R. Pelicer^{1,a}, D. P. Menezes¹, M. Dutra^{2,3}, O. Lourenço^{2,3}

SRC-HTM can significantly
reduce the core-crust
transition density, and can
even make some pasta-phases
disappear in the crust of NSs

$$\mathcal{U}_{\text{ther}}(\rho, \delta) \rightarrow 0$$



SRC-HMT on proton threshold of Urca process

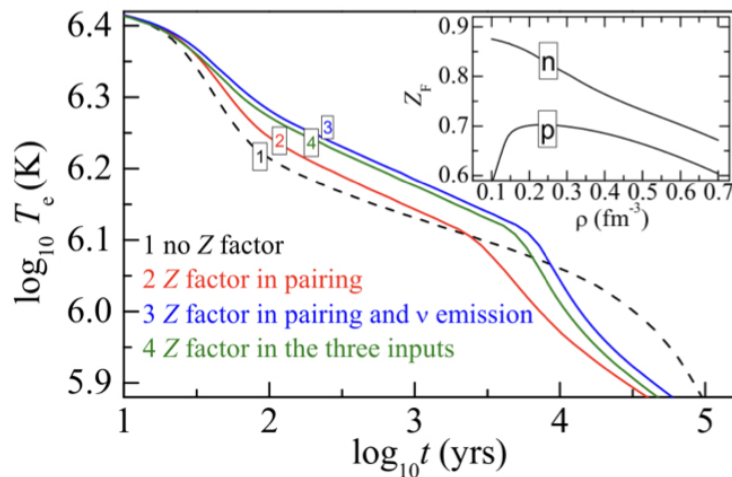


Urca process: $n \rightarrow p + e^- + \bar{\nu}_e$ HMT cutoff: $0 \leq t_j \leq \phi_j$

momentum consevation: $k_F^n (1 + t_n) \leq k_F^p (1 + t_p) + k_F^e$

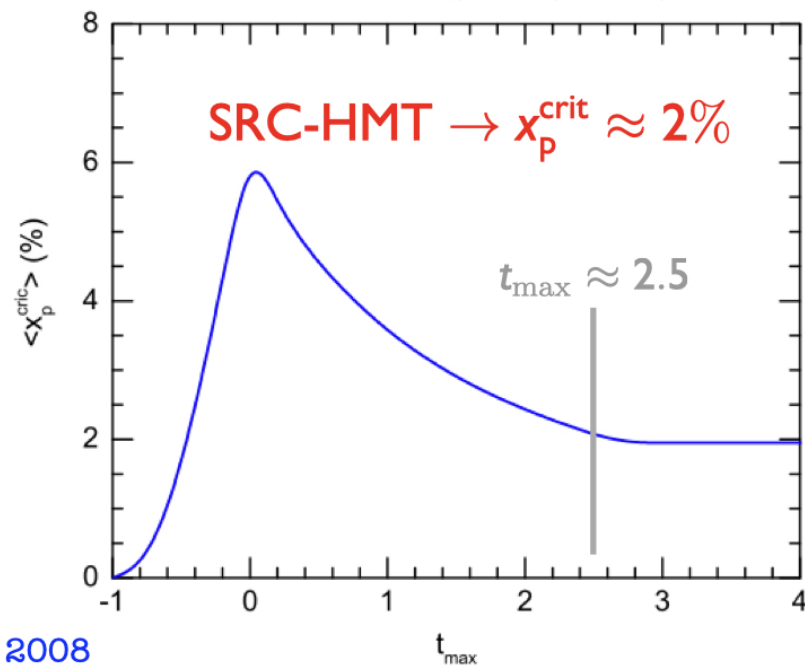
neutrality $k_F^p = k_F^e \rightarrow k_F^n (1 + t_n) \leq k_F^p (2 + t_p)$

$$\rightarrow x_p \geq x_p^{\text{crit}} = \frac{(1 + t_n)^3}{(1 + t_n)^3 + (2 + t_p)^3} \xrightarrow[t_n=t_p=0]{\text{free Fermi gas}} \frac{1}{9} \approx 11\%$$



see, more refs., e.g.,
Frankfurt et al., IJMPE, 2008
J.M. Dong et al., ApJ, 2016
Souza et al., 2020
Y.P. Wang, PRC, 2024
Sedrakian, 2024

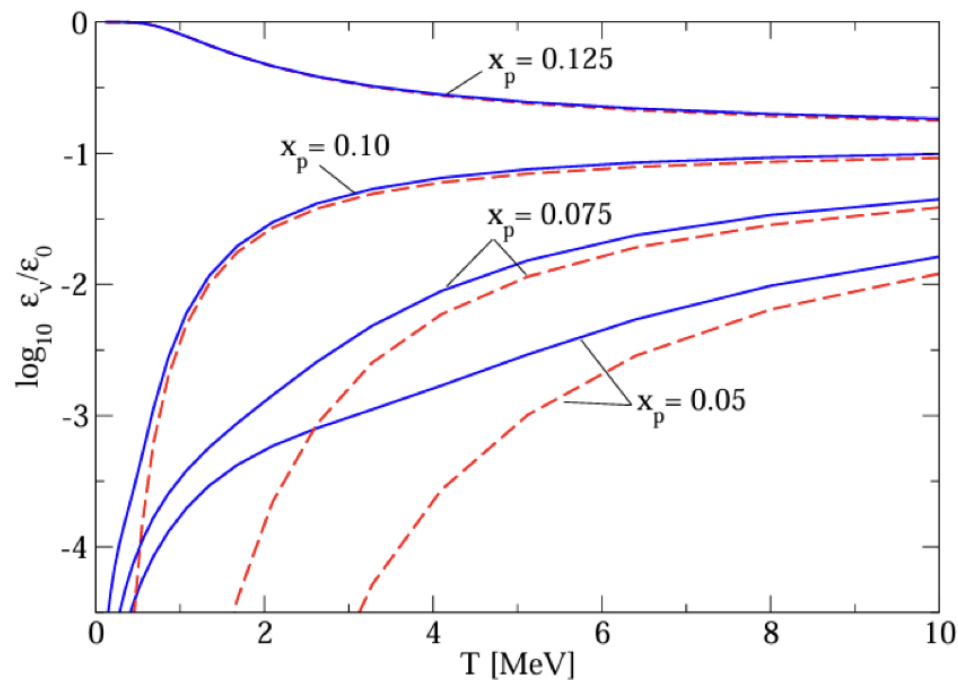
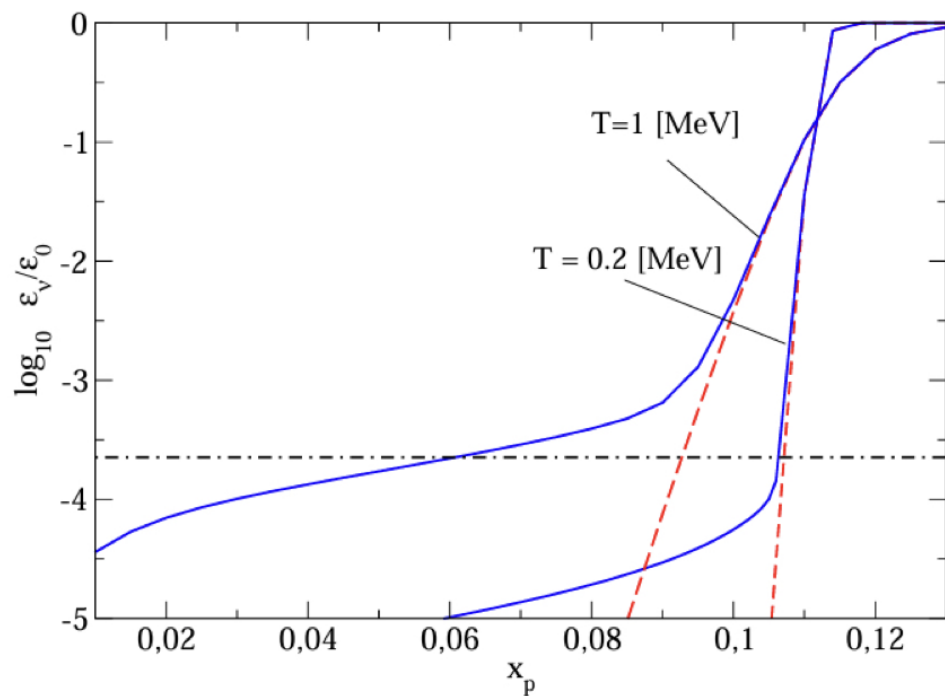
B.A. Li, et al., PPNP, 2018



SRC-HMT may effectively influence the cooling process (neutrino emission) in NSs

Neutrino emissivity due to SRCs

SRCs replace the sharp proton fraction threshold for direct Urca with a smooth onset, enhance the neutrino emissivity at low temperatures, and potentially alter NS cooling.



Sedrakian, PRL, 2024

Summary of my talk

- (1) k^{-4} HMT, $x_{\text{SNM}} \approx 25\%$, $x_{\text{PNM}} \lesssim 2\%$, isospin-dependent
 - (2) ps move faster than ns in neutron skins, $\Delta r_{\text{np}} \leftrightarrow \Delta E_{\text{pn}}^{\text{kin}}$
 - (3) HMT makes $E_{\text{sym}}^{\text{kin}}(\rho) < 0$ and softens $E_{\text{sym}}(\rho)$ at large ρ
 - (4) HMT affects NS properties, $x_{\text{p}}^{\text{crit}}$, $\rho_{\text{t}}^{\text{cc}}$, M_{NS} , ϵ_{ν} , pasta
 - (5) $k^{-\sigma}$ with $\sigma \geq 4$ for large k : $\delta E_0^{\text{kin}}(\rho_0) \approx 4.1\text{-}20.2 \text{ MeV}$
- * size/shape/isospin dependence of HMT $\leftrightarrow$ self-consistency
 - * dynamics (particle creation in HICs, ν -transport in NSs)
 - * $L \approx 27 \text{ MeV}$, optical model, PREX, $\dots$, what is missing?

Thank the organizer
for organizing this
wonderful workshop!

Thank you!

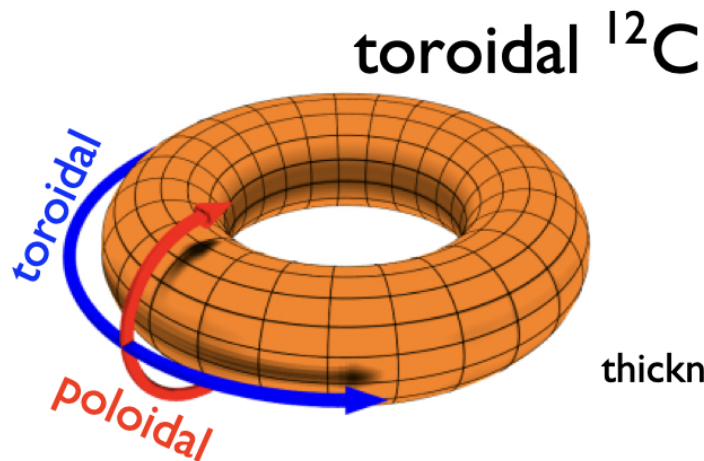
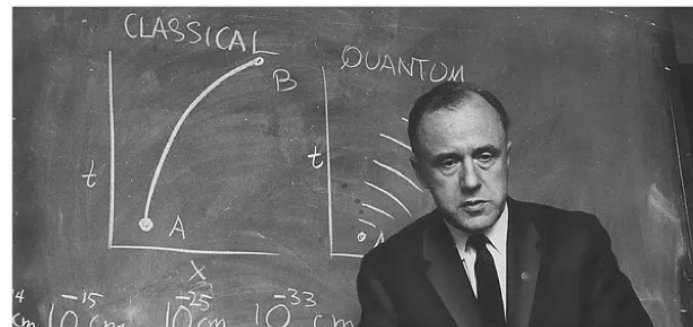
Low-dimensional nuclear/atomic systems



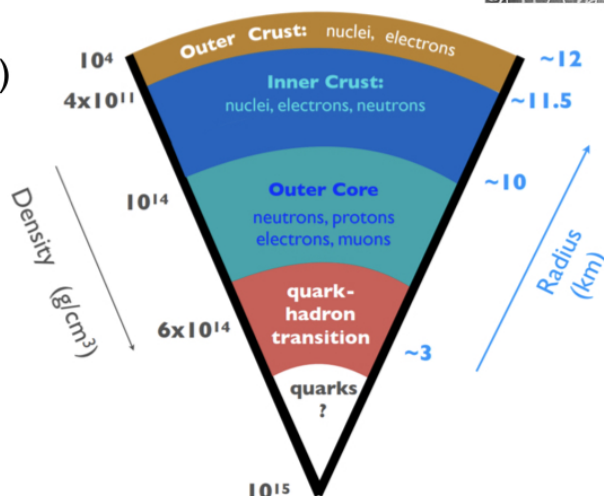
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examples:

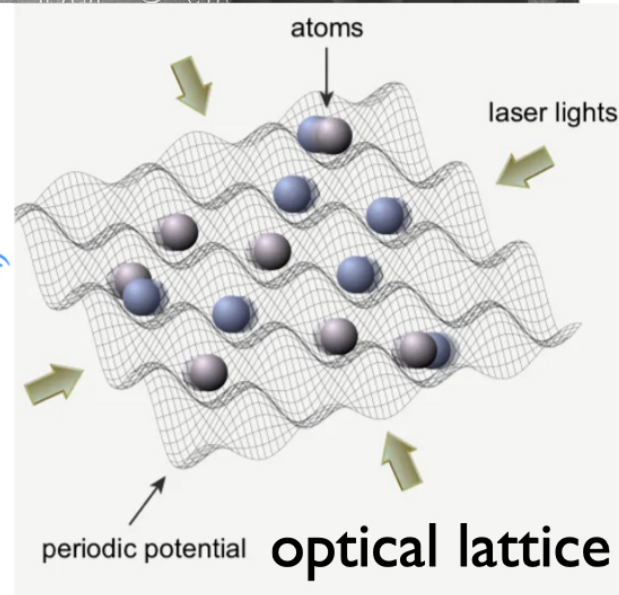
- (1) toroidal/spherical bubble topologies
(Wheeler, Siemens, Wong, 1960s-1970s; Natowitz, 2010s)
- (2) NS crust?
- (3) ultracold 1D/2D Fermion/Boson systems
(optical lattice/box; Feshbach resonance, ...)



Wheeler, 1963; C.Y. Wong, PLB, 1972



$$\text{thickness } t \approx (1-2) \text{ km} \rightarrow t/4\pi R^2 \sim 10^{-3} \text{ km}^{-1}$$



Example: Bertsch parameter from the epsilon expansion

$$\text{EOS} = \xi \frac{3}{5} \frac{k_F^2}{2M}: \text{Bertsch parameter } \xi?$$

(unitarity condition: $|k_F a| \rightarrow \infty$)

(Lee-Yang expansion via $|k_F a| \ll 1$ breaks down)

$$\frac{E}{N} = \frac{\hbar^2 k_F^2}{2m} \left[\frac{3}{5} + \frac{2}{3\pi} k_F a + \frac{4}{35\pi^2} (11 - 2 \ln 2) (k_F a)^2 + 0.23 (k_F a)^3 + \dots \right]$$

$D=4$ is non-interacting $\rightarrow$ extrapolation to $D=4-1=3$

PRL 97, 050403 (2006)

PHYSICAL REVIEW LETTERS

week ending
4 AUGUST 2006

ϵ Expansion for a Fermi Gas at Infinite Scattering Length

Yusuke Nishida^{1,2} and Dam Thanh Son²

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²Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195-1550, USA

(Received 2 May 2006; published 4 August 2006)

$$\epsilon = 4 - D \rightarrow \xi \approx \frac{1}{2} \epsilon^{3/2} + \frac{1}{16} \epsilon^{5/2} \ln \epsilon - 0.0246 \epsilon^{5/2} \xrightarrow{\epsilon=1} 0.378$$

Arnold et al., PRA, 2007

Ku et al., Science, 2012

Forbes et al., PRL, 2011

exp.: $\xi \approx 0.376$; QMC: $\xi \approx 0.383$

THE RENORMALIZATION GROUP AND THE ϵ EXPANSION*

Kenneth G. WILSON

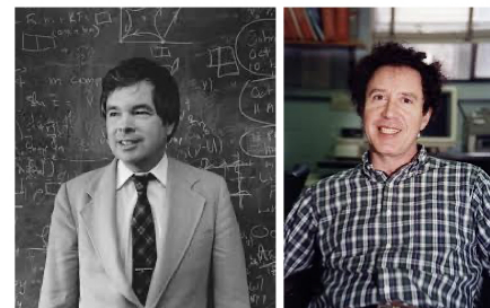
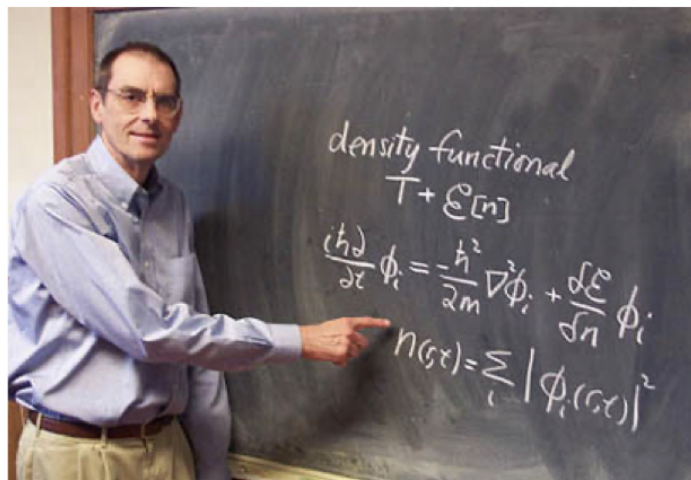
Institute for Advanced Study, Princeton, N.J. 08540, USA

and Laboratory of Nuclear Studies, Cornell University, Ithaca, N.Y. 14850, USA†

and

J. KOGUT‡

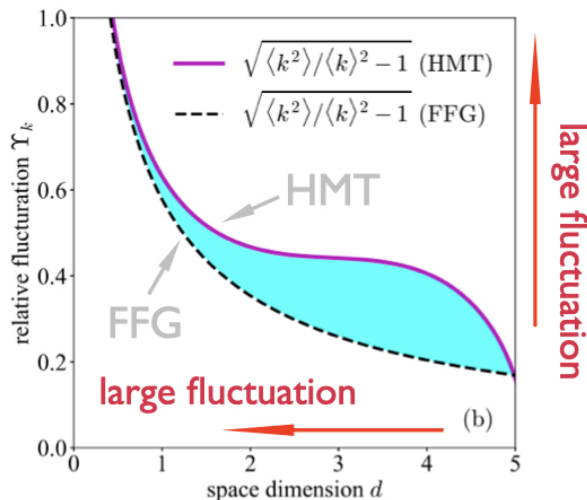
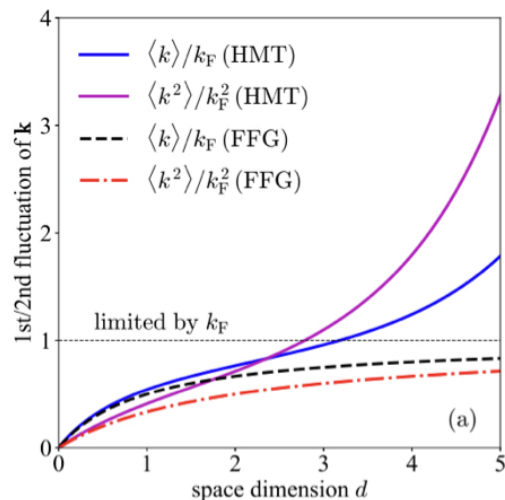
Institute for Advanced Study, Princeton, N.J. 08540, USA



Low-dimensional nuclear matter w/w.o SRC-HMT



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EQUATION OF STATE OF NEUTRON-RICH MATTER IN d -DIMENSIONS

Bao-Jun Cai¹

Quantum Machine Learning Laboratory, Shadow Creator Inc., Shanghai 201208, China

Bao-An Li²

Department of Physics and Astronomy, Texas A&M University-Commerce, Commerce, TX 75429-3011, USA

We made a first systematic investigation on the nuclear matter EOS in general D dimensions (including the ϵ -expansion for the EOS).

$$\text{relative fluctuation: } \Upsilon_k(d) = \sqrt{\frac{\langle |\mathbf{k}|^2 \rangle}{\langle |\mathbf{k}| \rangle^2} - 1}$$

some features:

- (1) free Fermi gas: momentum scale $\leq k_F$
- (2) SRC-HMT could generate large $|\mathbf{k}| > k_F$
- (3) large Υ_k for low $D \leftrightarrow$ Mermin-Wagner theorem (solid state physics)

- (4) low $D \rightarrow$ nearly free $\leftrightarrow$ extrapolation $\epsilon = 3 - D_{\text{free}}$

$$E_{\text{sym}}(\rho) = \frac{\bar{k}_F^2}{6M} + \frac{\bar{k}_F}{6} \frac{\partial U_0}{\partial |\mathbf{k}|} \Big|_{|\mathbf{k}|=\bar{k}_F} + \frac{1}{2} U_{\text{sym}}(\rho, \bar{k}_F) + \epsilon \left[\frac{\bar{k}_F^2}{6M} \left(2\sigma - \frac{1}{3} \right) + \frac{\bar{k}_F}{6} \left(\sigma \bar{k}_F \frac{\partial^2 U_0}{\partial |\mathbf{k}|^2} + \left(\sigma - \frac{1}{3} \right) \frac{\partial U_0}{\partial |\mathbf{k}|} \right) + \frac{\sigma}{2} \frac{\partial U_{\text{sym}}}{\partial |\mathbf{k}|} \cdot \bar{k}_F \right] \Big|_{|\mathbf{k}|=\bar{k}_F}$$

B.J. Cai and B.A. Li, Ann. Phys., 2022